



Katherine Semar Multiplication Calculation Policy

Progression through Calculations for Multiplication

Multiplication

EYFS

Before doubling can be introduced, children need to have a secure knowledge of counting, number facts and addition. Children are then introduced to the concept of doubling through practical games and activities and orally, including the use of the outdoor areas. Children act out 'doubling' by physically adding two equal groups together to find out the 'doubles' answer.

Links to ELG:

Automatically recall number bonds up to five and some number bonds to ten including double facts.

Explore and represent patterns within numbers up to ten, including even and odds, double facts and how quantities can be distributed equally.

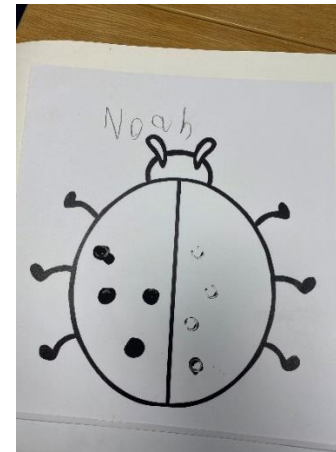
**Methods - Concrete, Pictorial,
Abstract (CPA)**

Concrete

Pictorial

Abstract

Doubling



Year 1

Multiplication Learning Objectives

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher

Guidance (non-statutory)

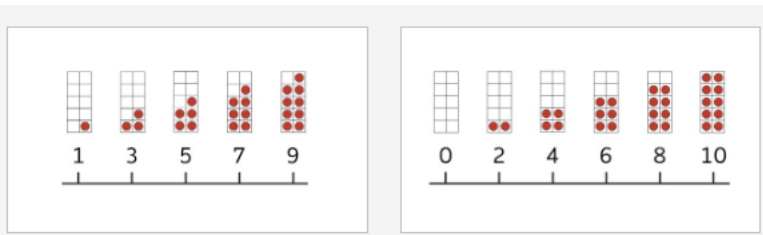
- Through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; and finding simple fractions of objects, numbers and quantities.
- They make connections between arrays, number patterns, and counting in 2s, 5s and 10s.

Mental Strategies

Children should experience [regular counting](#) on and back from different numbers in 1s and understand that the number is either increasing or decreasing by 1. Children should also skip count in multiples of 2, 5 and 10.

Children should memorise and reason with numbers in 2, 5 and 10 times tables.

They should see ways to represent odd and even numbers. This will help them to understand the pattern in numbers.



Children should begin to understand multiplication as scaling in terms of double and half. (e.g. that tower of cubes is double the height of the other tower)

Formal Written Calculations and Strategies

Understand multiplication is related to doubling and combining groups of the same size (repeated addition). Using concrete materials to make groups of 2, 5 and 10. Children to draw round each group.



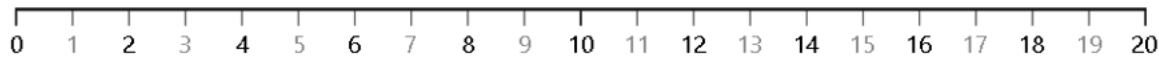
1 group of 2.... 2 groups of 2 3 groups of 2...

$$2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 16$$

$$2 \times 8 = 16$$

2 multiplied by 8.

There are 8 groups of 2.



Problem solving with concrete objects (including money and measures).

Use Cuisenaire and bar method to develop the vocabulary relating to 'times' –

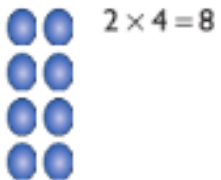
Pick up five, 4 times.

Use arrays to understand multiplication can be done in any order (commutative).



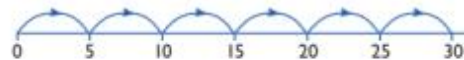
$$4 \times 2 = 8$$

$$2 \times 4 = 8$$



$$2 \times 4 = 8$$

$$4 \times 2 = 8$$



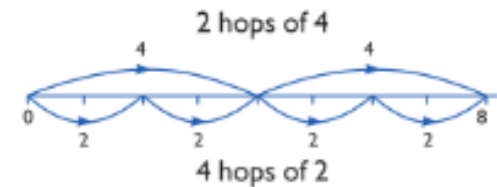
$$5 + 5 + 5 + 5 + 5 + 5 = 30$$

$$5 \times 6 = 30$$

5 multiplied by 6

6 groups of 5

6 hops of 5



Generalisation – identifying patterns

Understand 6 counters can be arranged as 3+3 or 2+2+2

Understand that when counting in twos, the numbers are always even.

Key Questions

Why is an even number an even number?

What do you notice?

What's the same? What's different?

Can you convince me?

How do you know?

Prove it!

Methods -
Concrete,
Pictorial,
Abstract
(CPA)

Concrete

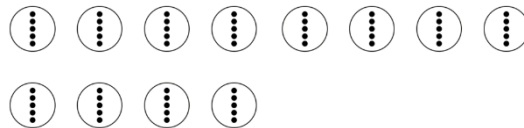
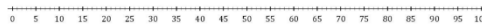
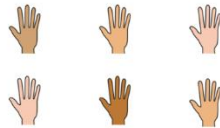
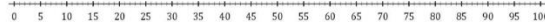
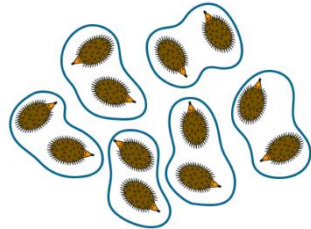
Pictorial

Abstract

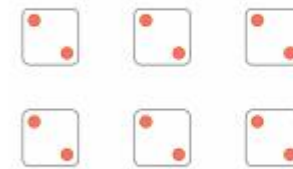
Count in
multiples
of 2, 5
and 10

Children to use counters or cubes to represent the problem

How many hedgehogs are there?
Count in groups of two.

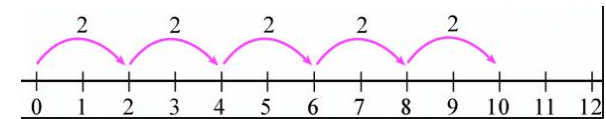


Count in visual then hidden groups of 2, 5 and 10.

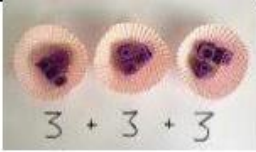


Orally counting out loud.

Counting using a counting stick – forwards and backwards from any given number.

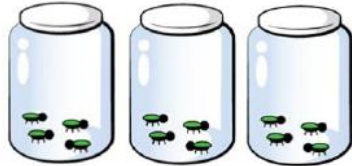


Repeated Addition



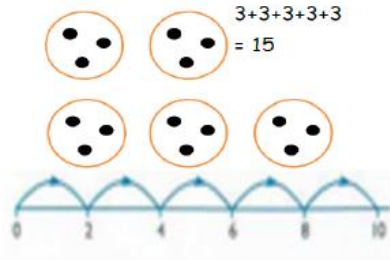
Use different objects to add equal groups.

3×4
 $4 + 4 + 4$
 There are 3 equal groups, with 4 in each group.

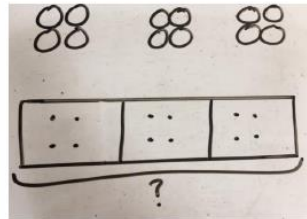


3 lots of 4
 3 groups of 4

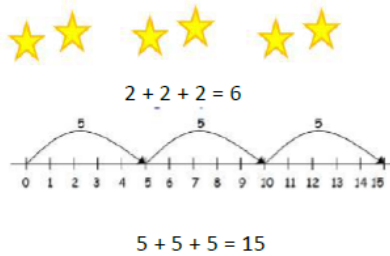
Use pictorial including number lines to solve prob
 There are 3 sweets in one bag.
 How many sweets are in 5 bags altogether?



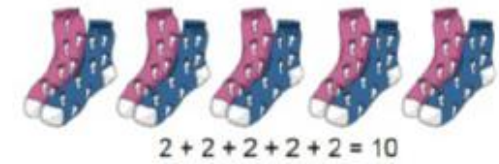
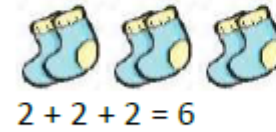
Children to represent the practical resources in a picture and use a bar model.



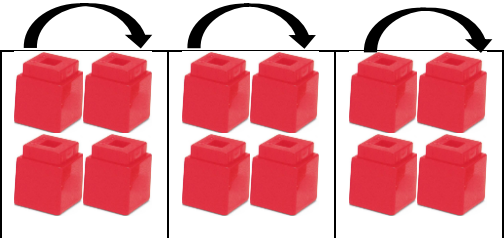
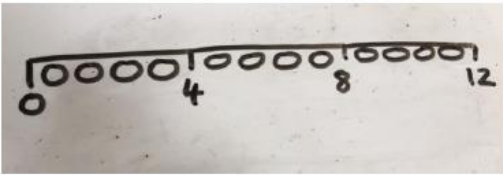
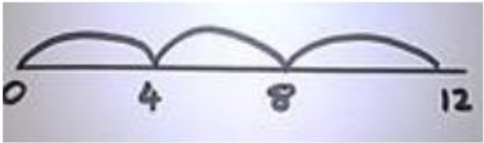
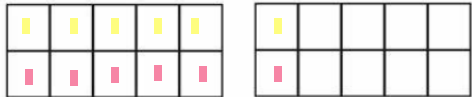
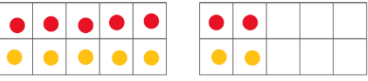
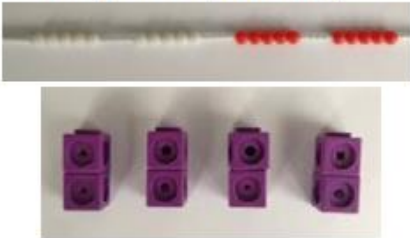
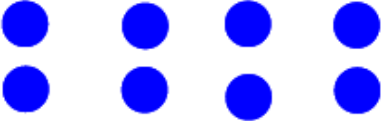
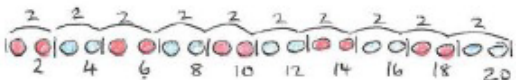
There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?

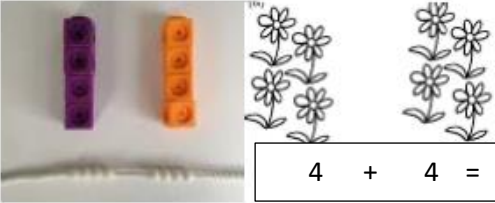

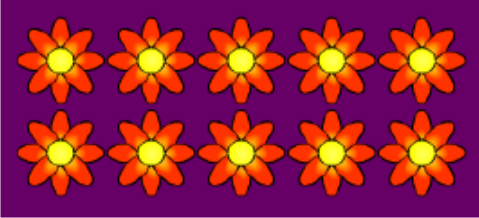
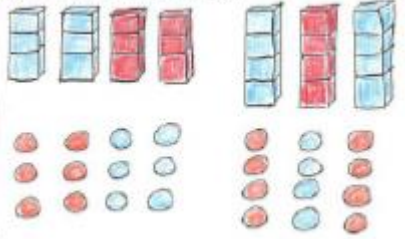


Write addition sentences to describe objects and pictures.



$2 + 2 + 2 + 2 + 2 = 10$
 $5 \times 2 = 10$

<p>Number line to show repeated groups</p>	<p>4 8 12</p> 	<p>Represent this pictorially alongside a number line e.g.:</p> 	<p>Abstract number line showing three jumps of four.</p> $3 \times 4 = 12$ 
<p>Doubling</p>	<p>Use concrete materials such as tens frames.</p>  $5 + 5 = 10$ $1 + 1 = 2$ $6 + 6 = 12$	<p>Children to draw their own doubles.</p>  <p>Double 7 is _____</p>	$4 + 4 =$ $5 + 5 =$ $6 + 6 =$
<p>Counting in Multiples</p>	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting.</p> 	 <p>Children make representations to show counting in multiples.</p> 	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> $2, 4, 6, 8, 10$ $5, 10, 15, 20, 25, 30$

<p>Making equal groups and counting the total</p>	 <p>Use manipulatives to create equal groups.</p>	<p>Draw and make representations</p>  <p>3 lots of 4</p>	<p>Written calculations</p> $4 + 4 + 4 =$
<p>Understanding Arrays</p>	<p>Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.</p> 	<p>Draw representations of arrays to show understanding</p> 	<p>Written calculations</p> $3 \times 2 = 6$ $2 \times 5 = 10$

Year 2

- Multiplication Learning Objectives**
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
 - Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×) and equals (=) signs
 - Show that multiplication of 2 numbers can be done in any order (commutative) and division of 1 number by another cannot

- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts

Guidance (non-statutory)

- Pupils use a variety of language to describe multiplication and division.
- Pupils are introduced to the multiplication tables. They practise to become fluent in the 2, 5 and 10 multiplication tables and connect them to each other. They connect the 10 multiplication table to place value, and the 5 multiplication table to the divisions on the clock face. They begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations.
- Pupils work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition. They begin to relate these to fractions and measures (for example, $40 \div 2 = 20$, 20 is a half of 40). They use commutativity and inverse relations to develop multiplicative reasoning (for example, $4 \times 5 = 20$ and $20 \div 5 = 4$).

Mental Strategies

Children should count regularly, on and back, in steps of 2, 3, 5 and 10.

Number lines should continue to be an important image to support thinking, for example

Children should practise times table facts which are presented in a variety of ways:

$$2 \times 1 = ?$$

$$? = 2 \times 2$$

$$2 \times 3 = ?$$

$$20 = 2 \times ?$$

Use a clock face to support understanding of counting in 5s.

Use money to support counting in 2s, 5s, 10s, 20s, 50s

Formal Written Calculations and Strategies

Expressing multiplication as a number sentence using x. Using understanding of the inverse and practical resources to solve missing number problems.

$$7 \times 2 = \square$$

$$\square = 2 \times 7$$

$$7 \times \square = 14$$

$$14 = \square \times 7$$

$$\square \times 2 = 14$$

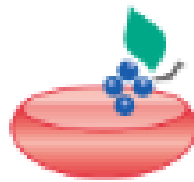
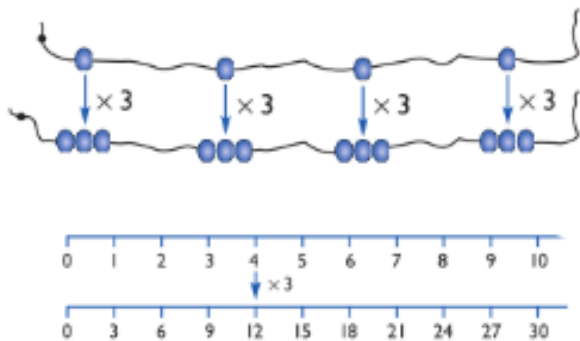
$$14 = 2 \times \square$$

$$\square \times \bigcirc = 14$$

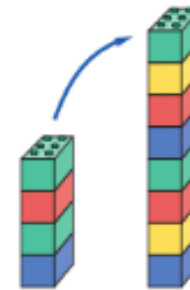
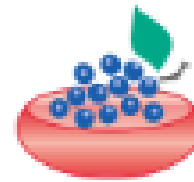
$$14 = \square \times \bigcirc$$

Develop understanding of multiplication using array and number lines (see Year 1). Include multiplications not in the 2, 5 or 10 times tables.

Begin to develop understanding of multiplication as scaling (3 times bigger/taller).



$$4 \times 3 = 12$$



double 4 is 8
 $4 \times 2 = 8$

Doubling numbers up to $10 + 10 = 2 \times 10$.

Link with understanding scaling.

Using known doubles to work out double 2digit numbers (double 15 = double 10 + double 5).

Towards written methods

$$5 \times 3 = 15$$

$$15 = 5 \times 3$$

$$80 = 8 \times 10$$

$$5 \times 6 = 10 \times 3$$

Generalisation – identifying patterns

Commutative law shown on array

Repeated addition can be shown mentally on a number line

Inverse relationship between multiplication and division. Use an array to explore how numbers can be organised into groups e.g.

$$2 \times 5 = 10$$

.....

.....

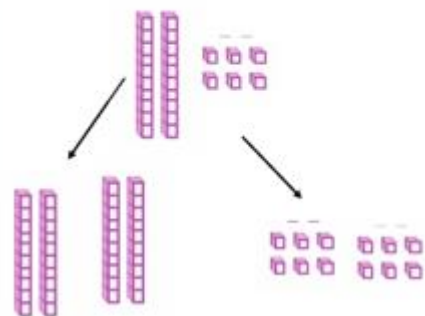
Key Questions

What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?

Methods - Concrete, Pictorial, Abstract (CPA)

Concrete

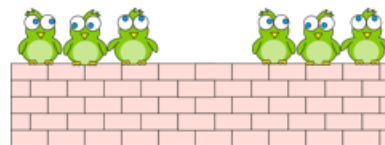
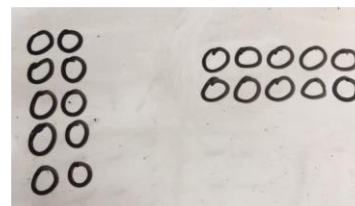
Model doubling using dienes and PV counters.



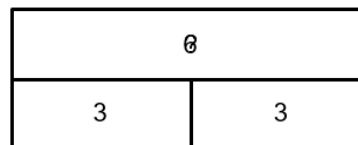
Double 20 = 40
Double 6 = 12
40 + 12 = 52

Pictorial

Draw pictures and representations to show how to double numbers

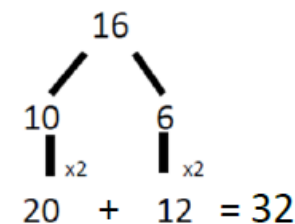


Double 3 is the same as 2 groups of 3.



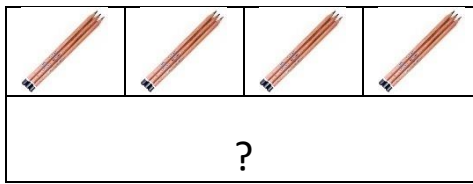
Abstract

Partition a number and then double each part before recombining it together.



Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)

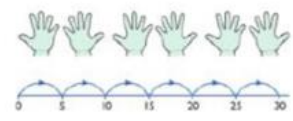
Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.



Bar models can be created with real life objects.

Bar models can now be used with pictorial representations.

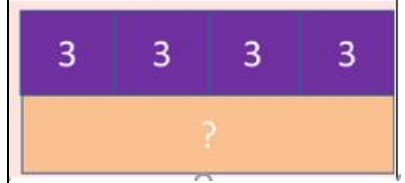
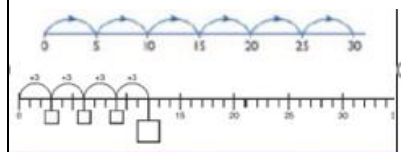
Number lines, counting sticks and bar models should be used to show representation of counting in multiples.



Count in multiples of a number aloud.

Write sequences with multiples of numbers.

0, 2, 4, 6, 8, 10



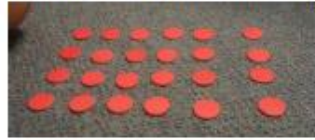
0, 3, 6, 9, 12, 15

0, 5, 10, 15, 20, 25, 30

Moving towards using written numbers in a bar model.

Understand Multiplication is Commutative

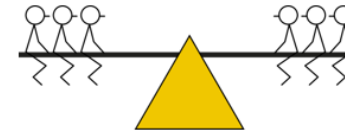
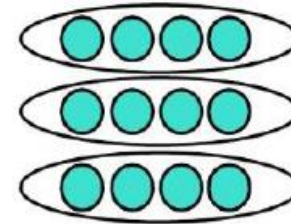
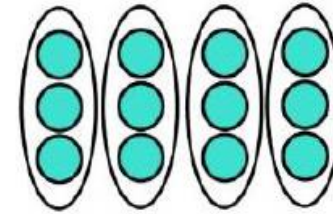
Create arrays using counters and cubes and Numicon.



Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.



Use representations of arrays to show different calculations and explore commutativity.



2 groups of 3 $2 \times 3 = 6$

3, two times $3 \times 2 = 6$

3, doubled $3 + 3 = 6$

Use an array to write multiplication sentences and reinforce repeated addition.



$$5 + 5 + 5 = 15$$

$$3 + 3 + 3 + 3 + 3 = 15$$

$$5 \times 3 = 15$$

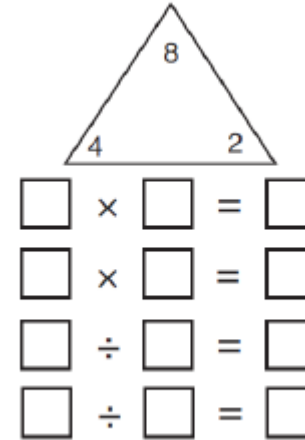
$$3 \times 5 = 15$$

Using the Inverse

This will be taught alongside division, so pupils learn how they work alongside each other.



Fact families



$$2 \times 4 = 8$$

$$4 \times 2 = 8$$

$$8 \div 2 = 4$$

$$8 \div 4 = 2$$

$$8 = 2 \times 4$$

$$8 = 4 \times 2$$

$$2 = 8 \div 4$$

$$4 = 8 \div 2$$

Show all 8 related fact families

Year 3

Multiplication Learning Objectives

- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects

Guidance (non-statutory)

- Pupils continue to practise their mental recall of multiplication tables when they are calculating mathematical statements in order to improve fluency. Through doubling, they connect the 2, 4 and 8 multiplication tables.
- Pupils develop efficient mental methods, for example, using commutativity and associativity (for example, $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$) and multiplication and division facts (for example, using $3 \times 2 = 6$, $6 \div 3 = 2$ and $2 = 6 \div 3$) to derive related facts ($30 \times 2 = 60$, $60 \div 3 = 20$ and $20 = 60 \div 3$).
- Pupils develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of short multiplication and division.
- Pupils solve simple problems in contexts, deciding which of the 4 operations to use and why. These include measuring and scaling contexts, (for example 4 times as high, 8 times as long etc) and correspondence problems in which m objects are connected to n objects (for example, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).

Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings and drawings to solve problems should be encouraged.

Children should practise times table facts which are presented in a variety of ways:

$3 \times 1 = ?$

$? = 3 \times 2$

$3 \times 3 = ?$

$12 = ? \times 3$

Formal Written Calculations and Strategies

Missing number problems

Continue with a range of equations as in Year 2 but with appropriate numbers.

Mental methods

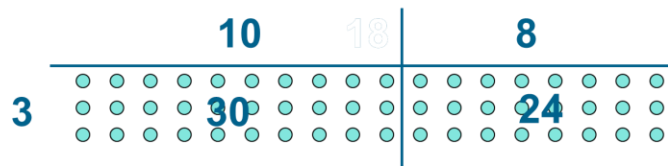
Doubling 2 digit numbers using partitioning

Demonstrating multiplication on a number line – jumping in larger groups of amounts

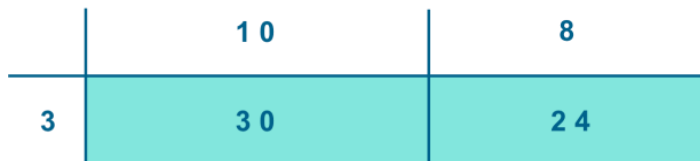
$13 \times 4 = 10 \text{ groups } 4 = 3 \text{ groups of } 4$

Written methods (progressing to 2digit x 1digit)

Developing written methods using understanding of visual images



Develop onto the grid method



Give children opportunities for children to explore this and deepen understanding using Dienes apparatus and place value counters.

Moving onto the formal written method.

Children to record what it is they are doing to show understanding.

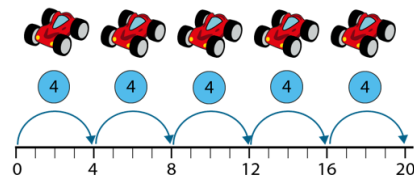
$$\begin{array}{l} 3 \times 23 \\ \swarrow \searrow \\ 20 \quad 3 \end{array} \quad \begin{array}{l} 3 \times 20 = 60 \\ 3 \times 3 = 9 \\ 60 + 9 = 69 \end{array}$$

$$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$$

Generalisation – identifying patterns

Connecting $\times 2$, $\times 4$ and $\times 8$ through multiplication facts

How many wheels? Count in groups of 4.



There are 20 wheels.

$$5 \times 4 = 20$$

$$4 \times 5 = 20$$

Comparing times tables with the same times tables which is ten times bigger. If $4 \times 3 = 12$, then we know $4 \times 30 = 120$. Use place value counters to demonstrate this.

When they know multiplication facts up to $\times 12$, do they know what $\times 13$ is? (i.e. can they use 4×12 to work out 4×13 and 4×14 and beyond?)

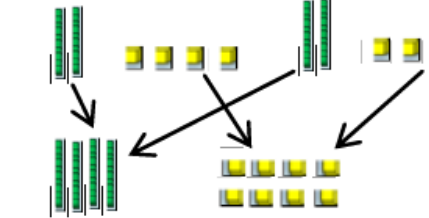
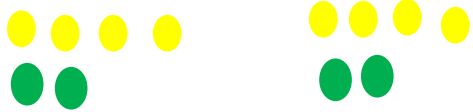
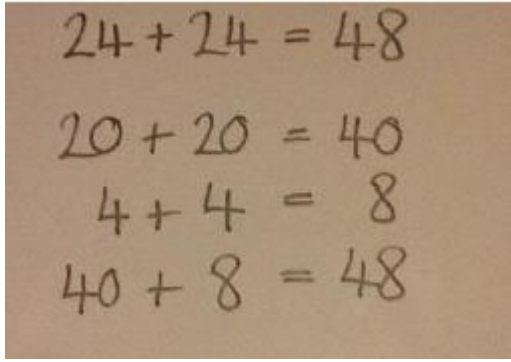
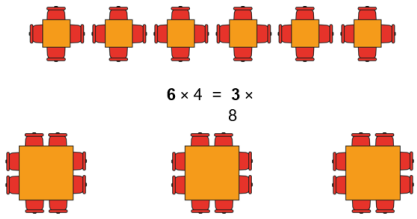
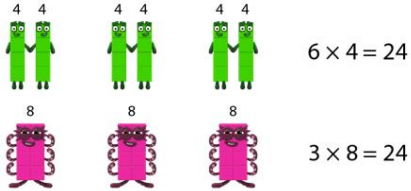
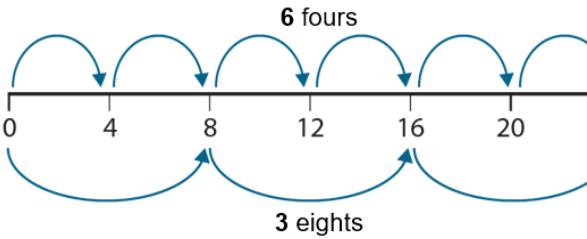
Key Questions

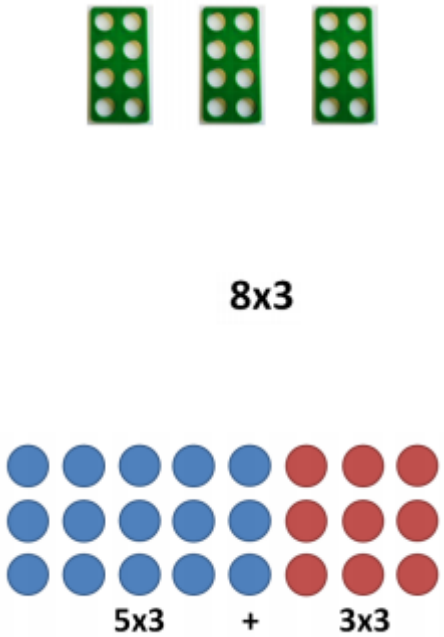

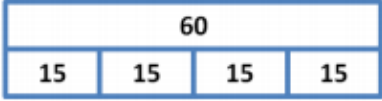

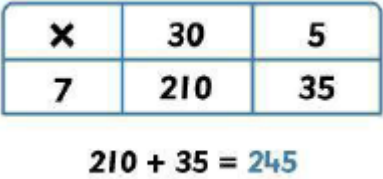
What do you notice?

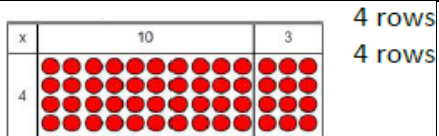
What's the same? What's different?

Can you convince me?

How do you know?

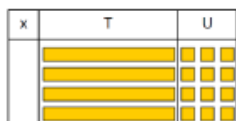
Methods - Concrete, Pictorial, Abstract (CPA)	Concrete	Pictorial	Abstract																		
<p>calculate doubles of 2-digit numbers through partitioning</p>	<p>Double 24 = 24 + 24 = 48</p> 	<p>Children draw representations of their own to show partitioning:</p> 																			
<p>Missing Numbers</p>	<p>Counters or multilink can be used to represent the chairs at the table</p> 	 	<p>24 = <input type="text"/> x <input type="text"/></p> <p>24 = <input type="text"/> x <input type="text"/></p> <table border="1" data-bbox="1482 901 2049 1085"> <tr> <td colspan="6">24</td> </tr> <tr> <td>4</td> <td>4</td> <td>4</td> <td>4</td> <td>4</td> <td>4</td> </tr> <tr> <td colspan="2">8</td> <td colspan="2">8</td> <td colspan="2">8</td> </tr> </table>	24						4	4	4	4	4	4	8		8		8	
24																					
4	4	4	4	4	4																
8		8		8																	

<p>Understand the inverse relationship between \times and \div; know \times as repeated adding, use to derive related multiplication facts.</p>	<p>Manipulatives such as coloured counters can be used to show multiplication as repeated addition. 2-colour arrays show distributive law.</p>  <p style="text-align: center;">8×3</p> <p style="text-align: center;">$5 \times 3 + 3 \times 3$</p>	<p>A range of images show multiplication as repeated addition. 2-colour arrays show distributive law.</p> 	<p>Bar model shows link between multiplication and division, and model division as sharing and grouping.</p> <p style="text-align: center;">$60 \div 4 = 15$</p>  <p style="text-align: center;"><i>'60 in four equal parts'</i></p> <p style="text-align: center;">$28 \div 7 = 4$</p>  <p style="text-align: center;"><i>'How many 7s in 28?'</i></p>
<p>Use efficient formal written methods for multiplication – Grid Method</p>	<p>Show the links with arrays to first introduce the grid method.</p>	<p>Children can represent the work they have done with place value counters in a way that they understand.</p> <p>They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.</p>	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p>  <p style="text-align: center;">$210 + 35 = 245$</p> <p>Moving forward, using the formal written grid method.</p>



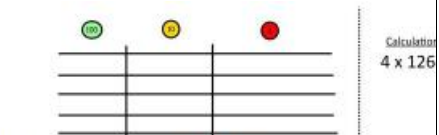
4 rows
4 rows

Move on to using Base 10 to move towards a more compact method.

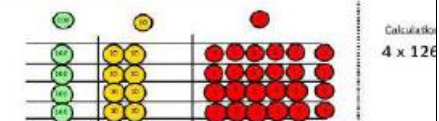


4 rows of 13

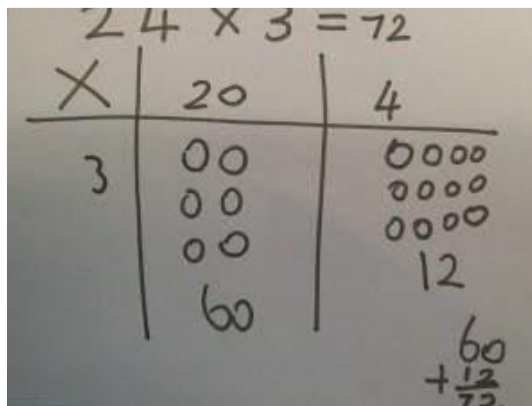
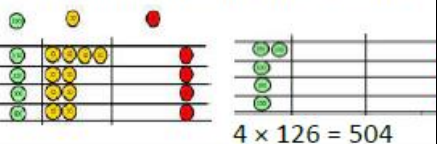
Move on to place value counters to show how we are finding groups of number. We are multiplying by 4 so need 4 rows.



Fill each row with 126.



Add up each column, starting with the ones making any exchanges needed.



$$\begin{array}{r} 34 \\ 3 \times \\ \hline 102 \\ 1 \end{array}$$

First multiply the ones
 $3 \times 4 = 12$
The ten then exchanged under the tens column.
Now multiply the tens and ones
 $30 \times 3 = 90$
Finally add the exchanged 10
 $90 + 10 = 100$

Children to record what it is they are doing to show understanding.

$$\begin{array}{r} 3 \times 23 \\ \swarrow \searrow \\ 20 \quad 3 \end{array} \quad \begin{array}{l} 3 \times 20 = 60 \\ 3 \times 3 = 9 \\ 60 + 9 = 69 \end{array}$$

$$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$$

Year 4

Multiplication Learning Objectives

- recall multiplication and division facts for multiplication tables up to 12×12
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout

- solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects
- recognise and show, using diagrams, families of common equivalent fractions
- count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten
- solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number

Guidance (non-statutory)

- Pupils continue to practise recalling and using multiplication tables and related division facts to aid fluency.
- Pupils practise mental methods and extend this to 3-digit numbers to derive facts, (for example $600 \div 3 = 200$ can be derived from $2 \times 3 = 6$).
- Pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers
- Pupils write statements about the equality of expressions (for example, use the distributive law $39 \times 7 = 30 \times 7 + 9 \times 7$ and associative law $(2 \times 3) \times 4 = 2 \times (3 \times 4)$). They combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations for example, $2 \times 6 \times 5 = 10 \times 6 = 60$.
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as the numbers of choices of a meal on a menu, or 3 cakes shared equally between 10 children.

Mental Strategies

Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits:

$$\square \times 5 = 160$$

Counting in multiples of 6, 7, 9, 25 and 1000, and steps of $1/100$.

Solving practical problems where children need to scale up. Relate to known number facts. (e.g. how tall would a 25cm sunflower be if it grew 6 times taller?).

Become fluent and confident to recall all tables to $\times 12$

Use the context of a week and a calendar to support the 7 times table (e.g. how many days in 5 weeks?)

Use of finger strategy for 9 times table.

Multiply 3 numbers together.

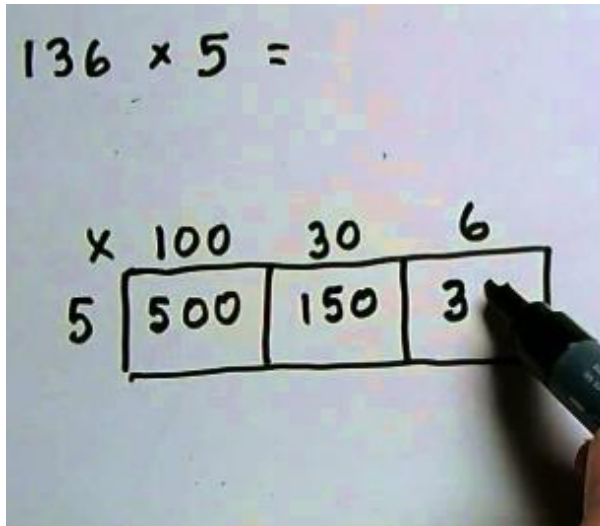
The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged.



They should be encouraged to choose from a range of strategies:

- Partitioning using $\times 10$, $\times 20$ etc
- Doubling to solve $\times 2$, $\times 4$, $\times 8$
- Recall of times tables
- Use of commutativity of multiplication

Formal Written Calculations and Strategies

Children to embed and deepen their understanding of the grid method to multiply 2digit \times 1digit and 3digit \times 1 digit. Ensure this is still linked back to their understanding of arrays and place value counters.



	Expanded	Standard
 4x6	36	36
	<u>x 4</u>	<u>x 4</u>
	24	144
 4x30	<u>120</u>	
	144	2

Children will learn the grid method to solve larger multiplication calculations to ensure understanding. They then move onto the expanded formal method. Once secure with this, they will learn the compact method (with exchanging).

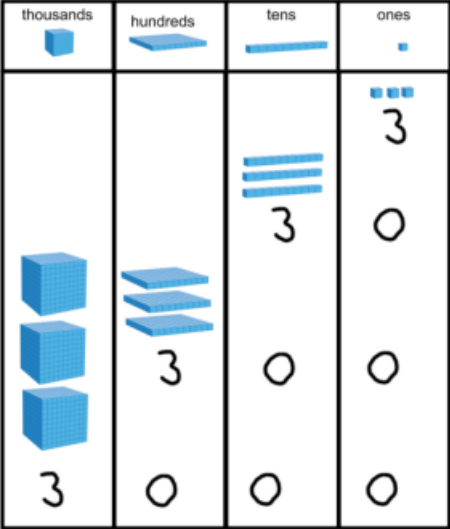
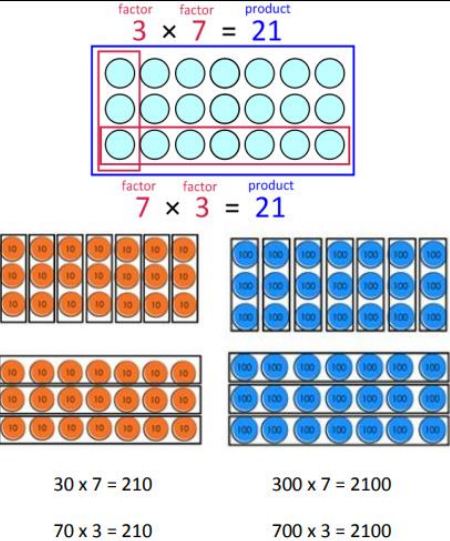
Generalisation – identifying patterns

Children given the opportunity to investigate numbers multiplied by 1 and 0.

When they know multiplication facts up to x12, do they know what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?)

Key Questions

- What do you notice?
- What's the same? What's different?
- Can you convince me?
- How do you know?

Methods - Concrete, Pictorial, Abstract (CPA)	<u>Concrete</u>	<u>Pictorial</u>	<u>Abstract</u>
Multiplying by 10 and 100		Children draw pictorial representations of their own e.g. of place value counters or Dienes (see concrete example).	$3 \times 10 = 30$ $3 \times 100 = 300$ $3 \times 1000 = 3000$
Using known facts and place value for mental multiplication involving multiples of 10 and 100		Children draw pictorial representations of their own e.g. of place value counters (see concrete example).	$30 \times 7 = 210$ 300 $70 \times 3 = 210$ 700 $7 \times 30 = 210$ 7 x 3 $3 \times 70 = 210$ 3 x 7
Mental multiplication of three 1- digit numbers, using the associative law	Questions should be presented in 'real life' contexts. Real objects can be used to introduce a problem e.g.		Written calculations: $(4 \times 2) \times 7$ or $4 \times (2 \times 7)$

Four pots each containing two flowers which each have seven petals. How many petals in total?

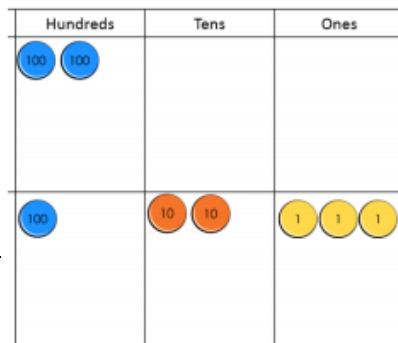
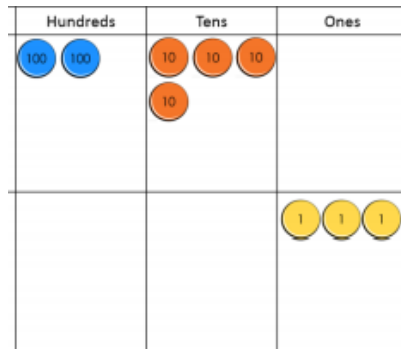
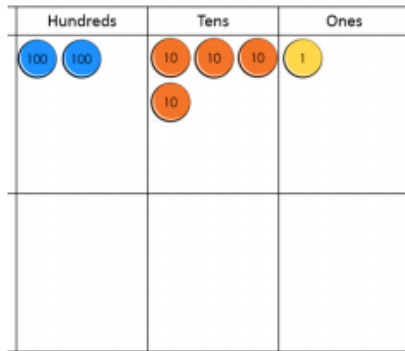
Pots of flowers could be used. Children can use coloured counters and pots to represent the question practically.



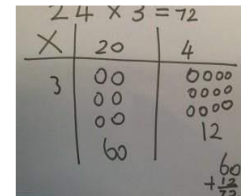
Pictorial representations also support the children to visualise the problem.

Multiply 2 and 3-digit numbers by a single digit, using all multiplication tables up to 12x12

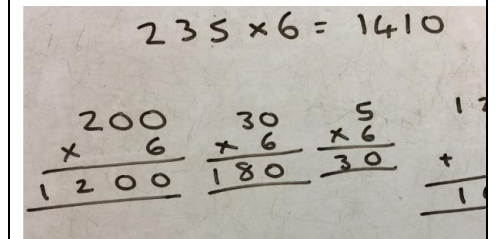
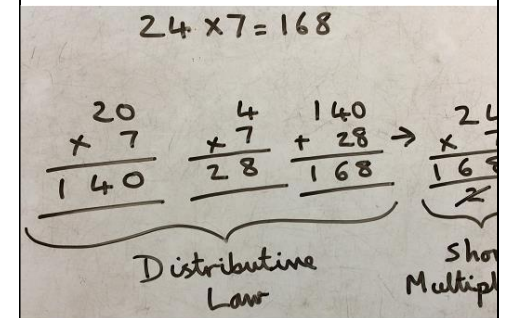
To calculate 241×3 , represent the number 241. Multiply each part by 3, regrouping as needed.



Children can represent the work they have done by drawing the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Expanded method should be revisited where necessary. However children should move onto the compact method.



$$400 \times 2 = 800$$

$$70 \times 2 = 140$$

$$3 \times 2 = 6$$

$$\hline 473 \times 2 = 946$$

$$\begin{array}{r}
 473 \\
 \times 2 \\
 \hline
 6 \\
 140 \\
 + 800 \\
 \hline
 946
 \end{array}$$

$$235 \times 6 = 1410$$

$$\begin{array}{r}
 235 \\
 \times 6 \\
 \hline
 1410 \\
 \hline
 23
 \end{array}$$

Year 5

Multiplication Learning Objectives

- identify multiples and factors, including finding all factor pairs of a number, and common factors of 2 numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally, drawing upon known facts

- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000
- recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3)
- solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates
- compare and order fractions whose denominators are all multiples of the same number
- identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths
- recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, $2 + \frac{5}{4} = 2\frac{5}{4} = 2\frac{1}{4} + 1 = 2\frac{1}{4} + 1$]
- multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams
- read and write decimal numbers as fractions [for example, $0.71 = \frac{71}{100}$]
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents ♣ round decimals with two decimal places to the nearest whole number and to one decimal place
- recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal
- solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{4}$ and those fractions with a denominator of a multiple of 10 or 25

Guidance (non-statutory)

- Pupils practise and extend their use of the formal written methods of short multiplication and short division. They apply all the multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations.
- They use and understand the terms factor, multiple and prime, square and cube numbers.
- Pupils interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (for example, $98 \div 4 = \frac{98}{4} = 24 \text{ r } 2 = 24\frac{1}{2} = 24.5 \approx 25$).
- Pupils use multiplication and division as inverses to support the introduction of ratio in year 6, for example, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1,000 in converting between units such as kilometres and metres
- Distributivity can be expressed as $a(b + c) = ab + ac$.
- They understand the terms factor, multiple and prime, square and cube numbers and use them to construct equivalence statements (for example, $4 \times 35 = 2 \times 2 \times 35$; $3 \times 270 = 3 \times 3 \times 9 \times 10 = 9^2 \times 10$).
- Pupils use and explain the equals sign to indicate equivalence, including in missing number problems (for example $13 + 24 = 12 + 25$; $33 = 5 \times ?$).

Mental Strategies

Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits.

Children should continue to count regularly, on and back, now including steps of powers of 10.

Multiply by 10, 100, 1000, including decimals (Moving Digits ITP)

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged.

They should be encouraged to choose from a range of strategies to solve problems mentally:

- Partitioning using $\times 10$, $\times 20$ etc
- Doubling to solve $\times 2$, $\times 4$, $\times 8$
- Recall of times tables
- Use of commutativity of multiplication
-

If children know the times table facts to 12×12 . Can they use this to recite other times tables (e.g. the 13 times tables or the 24 times table)

Use practical resources and jottings to explore equivalent statements (e.g. $4 \times 35 = 2 \times 2 \times 35$).

Recall of prime numbers up to 19 and identify prime numbers up to 100 (with reasoning).

Solving practical problems where children need to scale up. Relate to known number facts.

Formal Written Calculations and Strategies

Long multiplication using place value counters

Children to explore how the grid method supports an understanding of long multiplication (for 2digit \times 2digit)

Generalisation – identifying patterns

Relating arrays to an understanding of square numbers and making cubes to show cube numbers.

Understanding that the use of scaling by multiples of 10 can be used to convert between units of measure (e.g. metres to kilometres means to times by 1000)

Key Questions

What do you notice?

What's the same? What's different?

Can you convince me?

How do you know?

How do you know this is a prime number?

Year 6

Multiplication Learning Objectives

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the 4 operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $4 \frac{1}{2} \times 2 \frac{1}{2} = 8 \frac{1}{2}$]
- multiply one-digit numbers with up to two decimal places by whole numbers
- recall and use equivalences between simple fractions, decimals and percentages, including in different contexts

Guidance (non-statutory)

- Pupils practise addition, subtraction, multiplication and division for larger numbers, using the formal written methods of columnar addition and subtraction, short and long multiplication, and short and long division.
- They undertake mental calculations with increasingly large numbers and more complex calculations.
- Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency.
- Pupils round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50, etc, but not to a specified number of significant figures.
- Pupils explore the order of operations using brackets; for example, $2 + 1 \times 3 = 5$ and $(2 + 1) \times 3 = 9$.

Common factors can be related to finding equivalent fractions.

Mental Strategies

Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits.

Identifying common factors and multiples of given numbers

Solving practical problems where children need to scale up. Relate to known number facts.

Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. $20 - 5 \times 3 = 5$; $(20 - 5) \times 3 = 45$

They should be encouraged to choose from a range of strategies to solve problems mentally:

- Partitioning using x10, x20 etc

- Doubling to solve x2, x4, x8
- Recall of times tables
- Use of commutativity of multiplication
-

If children know the times table facts to 12 x 12. Can they use this to recite other times tables (e.g. the 13 times tables or the 24 times table).

Formal Written Calculations and Strategies

Continue to refine and deepen understanding of written methods including fluency for using long multiplication.

Generalisation – identifying patterns

Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as BODMAS, or could be encouraged to design their own ways of remembering.

Understanding the use of multiplication to support conversions between units of measurement.

Key Questions

What do you notice?

What's the same? What's different?

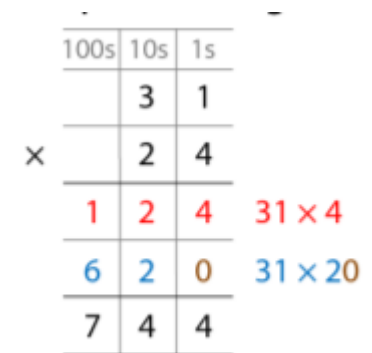
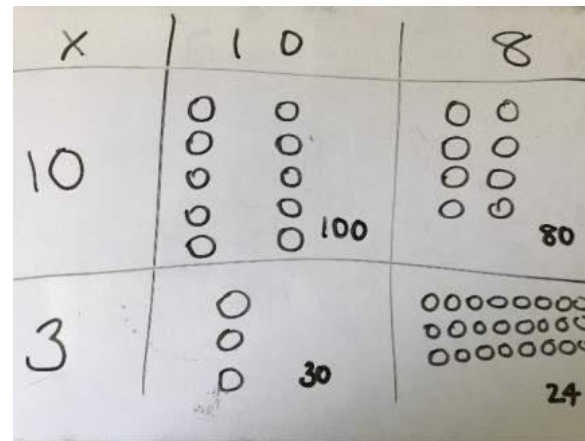
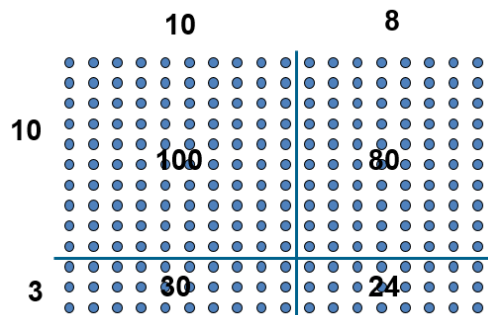
Can you convince me?

How do you know?

Year 5 and 6

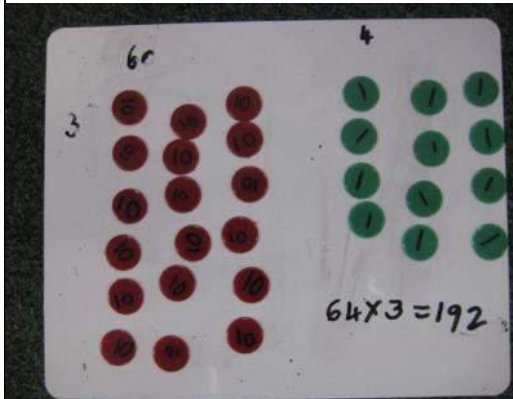
Expanded method

Show the link with arrays to first introduce the expanded method.



Compact method

Children can continue to be supported by place value counters at the stage of multiplication.

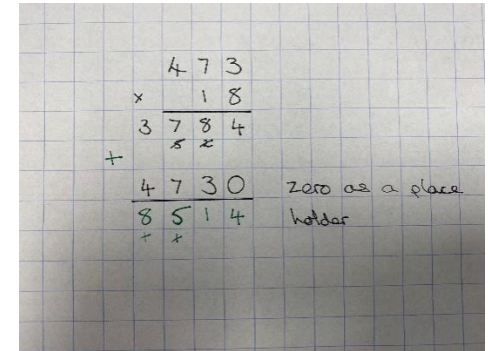


It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

12×34	34	
\times	$\times 12$	
\times	68	
\times	340	
\times	408	
\times	408	

243	Thousands	Hundreds	Tens	Ones	
$\times 12$					
486		100 100 100	10 10 10	1 1 1	
2430	1000 1000	100 100 100	10 10 10	1 1 1	243×2
					243×10

This moves to the more compact method.



Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

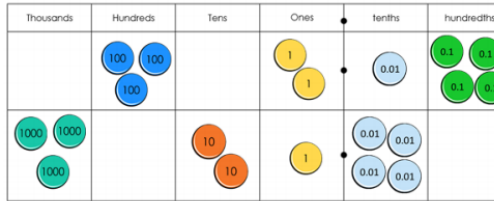
When you multiply by ten, each part is ten times greater. The ones become tens, the tens become hundreds, etc. When multiplying whole numbers, a

Children may draw their own pictorial representations (of PV counters) on a place value chart to support their understanding.

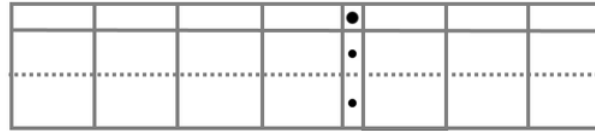
Children will then move onto using numbers and place value charts to support their calculations.

zero holds a place so that each digit has a value that is ten times greater.

$102.14 \times 10 = 1021.4$



ii) $4.617 \times 10 =$ _____

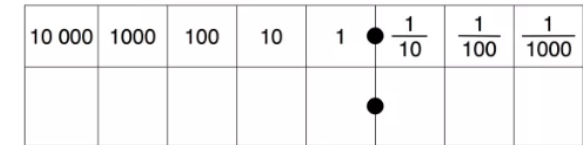
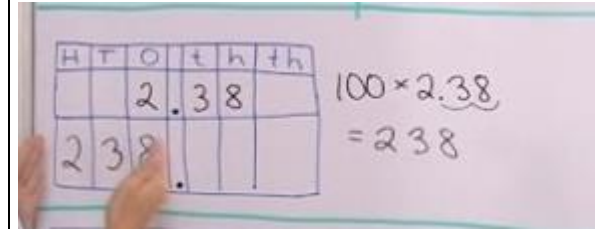
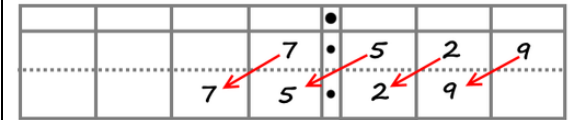


Th	H	T	O	.	t	h	th
			3	.	2	1	
		3	2	.	1		
	3	2	1	.			
3	2	1	0	.			
				.			

$3.21 \times 1000 = 3210$

$102.14 \times 10 = 1021.4$

i) $7.529 \times 10 = 75.29$

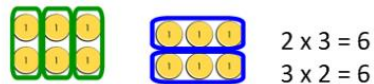


Multiplying

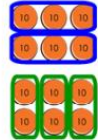
- X 10 digits move LEFT 1 space
- X 100 digits move LEFT 2 spaces
- X 1000 digits move LEFT 3 spaces



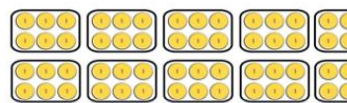
Using known facts and place value to derive multiplication facts



$2 \times 30 = 60$
 $30 \times 2 = 60$



$3 \times 20 = 60$
 $20 \times 3 = 60$



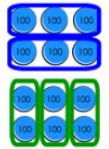
$2 \times 3 \times 10 = 60$

These are the multiplication facts pupils should be able to derive from a known fact:

2 100 000	700 000 x 3	70 000 x 30	7000 x 300	700 x 3000	70 x 30 000	7 x 300 000
210 000	70 000 x 3	7000 x 30	700 x 300	70 x 3000	7 x 30 000	
21 000	7000 x 3	700 x 30	70 x 300	7 x 3000		
2100	700 x 3	70 x 30	7 x 300			
210	70 x 3	7 x 30				
21	= 7 x 3					
2.1	0.7 x 3	7 x 0.3				
0.21	0.07 x 3	0.7 x 0.3	7 x 0.03			
0.021	0.007 x 3	0.07 x 0.3	0.7 x 0.03	7 x 0.003		

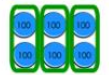
$$2 \times 300 = 600$$

$$300 \times 2 = 600$$



$$3 \times 200 = 600$$

$$200 \times 3 = 600$$



$$2 \times 30 \times 10 = 600$$

$$20 \times 3 \times 10 = 600$$



$$20 \times 30 = 600$$

$$30 \times 20 = 600$$

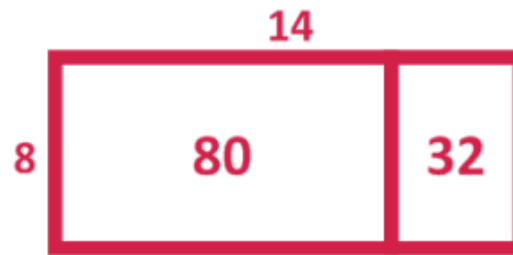
Multiply by partitioning one number and multiplying each part

Distributive law $a \times (b + c) = a \times b + a \times c$

$$8 \times 14 = 8 \times 10 + 8 \times 4$$

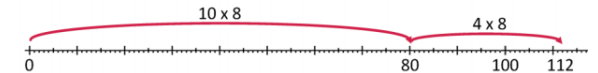


Cuisenaire rods to build array



Represent with area model

$$8 \times 14 = 8 \times 10 + 8 \times 4$$



Jottings on a number line

Using knowledge of factors

Calculate 6×24 or 12×12 by using factor pairs of 24. Two and twelve are factors of 24:

In Year 5 pupils are expected to be able to identify factor pairs and this knowledge can be used to calculate. Pupils will be using the commutative and associative laws of multiplication.

Commutative law $a \times b = b \times a$

Associative law $a \times b \times c = (a \times b) \times c = a \times (b \times c)$ They should explore and compare the different options and choose the most efficient order to complete calculations.

$$6 \times 2 \times 12$$



$$6 \times 12 \times 2$$



Three and eight are factors of 24:

$$6 \times 3 \times 8$$



$$6 \times 8 \times 3$$



Four and six are factors of 24:

$$6 \times 4 \times 6$$



$$6 \times 6 \times 4$$

