

Katherine Semar Subtraction Calculation Policy

Progression through Calculations for Subtraction

Subtraction

EYFS

Before subtraction can be introduced, children need to have a secure knowledge of number. Children will first learn to partition two or more numbers using a range of concrete apparatus to understand how a whole can be split into its constituent parts. Using talk and practical games, children will be able to apply factual fluency to solve problems. In other words, children should use their number bond knowledge to help with subtraction facts.

Children will progress to the concept of subtraction as reduction by learning that subtracting means taking away a certain number of objects from a group (leaving them with less objects). They will be introduced to the concept of subtraction through practical games and activities and act out subtractions to physically subtract a number of objects from a group and use arm gestures to represent the signs - and =.

Adults model subtraction vocabulary supported by age appropriate definition. An example of this is "subtraction means we take away objects from a group / we have got fewer objects now."

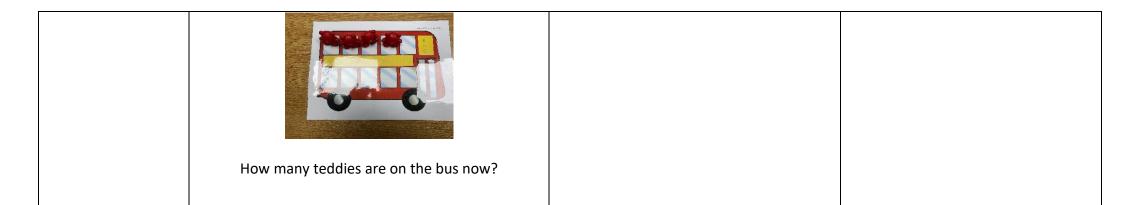
Links to ELG:

Have a deep understanding of numbers to ten, including the composition of each number.

Automatically recall number bonds up to five (including subtraction facts) and some bonds to ten including double facts.

Methods - Concrete, Pictorial, Abstract (CPA)	<u>Concrete</u>	<u>Pictorial</u>	<u>Abstract</u>
Partitioning This model develops knowledge of the inverse relationship between addition and subtraction and is used to find the	There are five teddies waiting at the bus stop.		

answer to missing number problems.	Three teddies get onto the bus. How many teddies are NOT on the bus? Children progress to one part being hidden and applying their knowledge of number answer the question.	Children are encouraged to represent their mathematical thinking in their own way.	
Reduction Use physical objects to demonstrate how something can be taken away. Move on to crossing out drawn representations. This can be developed by representing a group of ten with a line and ones with dots	First, there were seven teddies on the bus. Then, three teddies got off.	First, there were ten green bottles sitting on the wall. Then two fell off. How many are there now?	



- read, write and interpret mathematical statements involving subtraction (–) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- subtract one-digit and two-digit numbers to 20, including zero

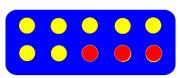
solve one-step problems that involve subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ? - 9.

Mental Strategies

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.

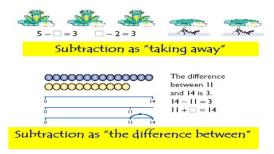
Children should memorise and reason with number bonds for numbers to 20, experiencing the = sign in different positions.

They should see addition and subtraction as related operations. E.g. 7 + 3 = 10 is related to 10 - 3 = 7, understanding of which could be supported by an image like this.



Use bundles of straws and Dienes to model partitioning teen numbers into tens and ones.

Children should begin to understand subtraction as both taking away and finding the difference between, and should find small differences by counting on.



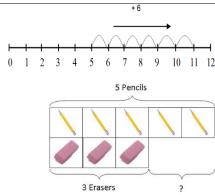
Formal Written Calculations and Strategies

Missing number problems e.g. $7 = \square$ - 9; 20 - \square = 9; 15 - 9 = \square ; \square - \square = 11; 16 - 0 = \square

Use concrete objects and pictorial representations. If appropriate, progress from using number lines with every number shown to number lines with significant numbers shown.

Understand subtraction as take-away:

Understand subtraction as finding the difference:

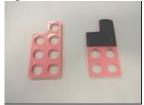


The above model would be introduced with concrete objects which children can move (including cards with pictures) before progressing to pictorial representation. The use of other images is also valuable for modelling subtraction e.g. Numicon, bundles of straws, Dienes apparatus, multi-link cubes, bead strings

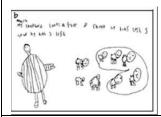
Children develop subtraction facts initially to ten and then to 20.

Record related number facts (and make links to related addition facts)

e.g. 9 - 4 = 5, 9 - 5 = 4



Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures etc. e.g. 8 sheep take away 5 sheep



Generalisation - identifying patterns

True or false? Subtraction makes numbers smaller

When introduced to the equals sign, children should see it as signifying equality. They should become used to seeing it in different positions.

Relating subtraction facts e.g.

5-3=2

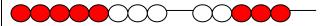
15-3=12

25-3=22

35-3=32

Etc.

Bead strings can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2. e.g. 13 - 5 = 8 (using 5 = 3 + 2) so 13 - 3 = 10 and 10 - 2 = 8



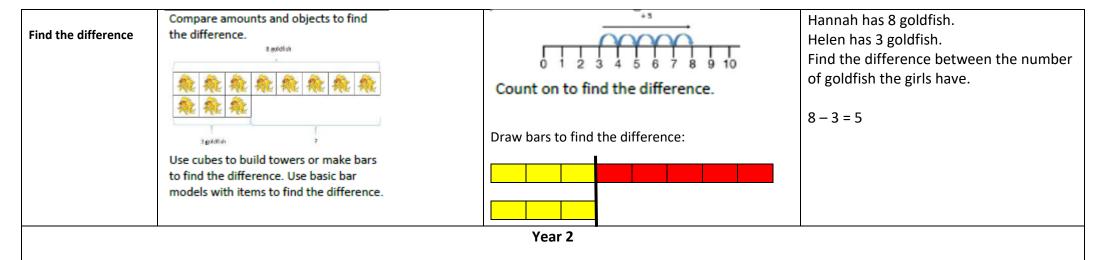
Key Questions

How many more to make...? How many more is... than...? How much more is...? How many are left/left over? How many have gone? One less, two less, ten less... How many fewer is... than...? How much less is...?

What can you see here?

Is this true or false?

Methods - Concrete,	<u>Concrete</u>	<u>Pictorial</u>	<u>Abstract</u>
Pictorial, Abstract			
(CPA)			
	Use physical objects, counters, cubes etc. to show	Cross out drawn objects to show what	4 - 2 = ?
Taking away	how objects can be taken away.	has been taken away.	4
ones			? – 2 = 2
		4-2=2	? = 4 - 2
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in	Count back on a number line or number track	Put 13 in your head, count back 4. What number do you have?
	ones.	9 10 11 12 13 14 15 Start at the bigger number and count back the smaller number, showing the jumps on the number line.	
	13 - 4 = 9		



- solve problems with subtraction:
- using concrete objects and pictorial representations, including those involving numbers, quantities and measures their increasing knowledge of mental and written methods

applying

- recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100
- subtract numbers using concrete objects, pictorial representations, and mentally, including:
- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

Mental Strategies

Children should count regularly, on and back, in steps of 2, 3, 5 and 10. Counting back in tens from any number should lead to subtracting multiples of 10.

Children should practise subtraction to 20 to become increasingly fluent. They should use the facts they know to derive others, e.g using 10 - 7 = 3 and 7 = 10 - 3 to calculate 100 - 70 = 30 and 70 = 100 - 30.

Children should learn to check their calculations, including by adding to check.

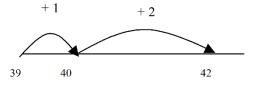
They should continue to see subtraction as both take away and finding the difference, and should find a small difference by counting up.

They should use Dienes to model partitioning into tens and ones and learn to partition numbers in different ways e.g. 23 = 20 + 3 = 10 + 13.

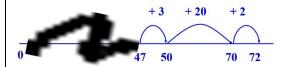
Formal Written Calculations and Strategies

Missing number problems e.g. $52 - 8 = \Box$; $\Box - 20 = 25$; $22 = \Box - 21$; $6 + \Box + 3 = 11$

It is valuable to use a range of representations (also see Y1). Continue to use number lines to model take-away and difference. E.g.



The link between the two may be supported by an image like this, with 47 being taken away from 72, leaving the difference, which is 25.



Children can use their understanding of patterning, place value and partitioning to derive number facts.

e.g. 7 - 3 = 4 (known fact)

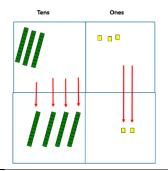
17 - 3 - 14

27 - 3 = 24



Towards written methods

Recording addition and subtraction in expanded columns can support understanding of the quantity aspect of place value and prepare for efficient written methods with larger numbers. The numbers may be represented with Dienes apparatus or place value counters. E.g. 75 – 42



Generalisation – identifying patterns

Noticing what happens when you count in tens (the digits in the ones column stay the same)

Odd - odd = even; odd - even = odd; etc

show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot

Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.



15 + 5 = 20

5 + 15 = 20

20 - 15 = 5

20 - 5 = 15

Key Questions

How many more to make...? How many more is... than...? How much more is...? How many are left/left over? How many fewer is... than...? How much less is...? Is this true or false?

If I know that 7 + 2 = 9, what else do I know? (e.g. 2 + 7 = 9; 9 - 7 = 2; 9 - 2 = 7; 90 - 20 = 70 etc).

What do you notice? What patterns can you see?

Methods - Concrete, Pictorial, Abstract (CPA)	Concrete	<u>Pictorial</u>	Abstract
Column method without exchanging/regrouping	75 – 42 = 33 Use Base 10 to make the bigger number then take the smaller number away.	Draw the Base 10 or place value counters alongside the written calculation to help to show working.	$47 - 24 = 23$ $-\frac{40 + 7}{20 + 3}$ This will lead to a clear written column subtraction.
	Show how you partition numbers to subtract. Again make the larger number first.	Tens Units Op Op Op Op Op Op Op O	-12-20
	Year 3	(10p) (10p)	

Year 3

- subtract numbers mentally, including:
- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds
- subtract numbers with up to three digits, using formal written methods of columnar subtraction
- estimate the answer to a calculation and use inverse operations to check answers

solve problems, including missing number problems, using number facts, place value, and more complex subtraction.

Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged.

Children should continue to partition numbers in difference ways.

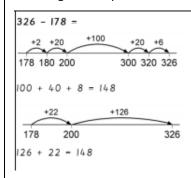
They should be encouraged to choose the mental strategies which are most efficient for the numbers involved, e.g. counting up (difference, or complementary addition) for 201 – 198; counting back (taking away / partition into tens and ones) for 201 – 12.

Calculators can usefully be introduced to encourage fluency by using them for games such as 'Zap' [e.g. Enter the number 567. Can you 'zap' the 6 digit and make the display say 507 by subtracting 1 number?]

The strategy of adjusting can be taken further, e.g. subtract 100 and add one back on to subtract 99. Subtract other near multiples of 10 using this strategy.

Formal Written Calculations and Strategies

With three-digit numbers the number of steps can again be reduced, enabling children to work out answers to calculations such as 326 – 178 first in small steps and then more compact by using knowledge of complements to 100 The most compact form of recording becomes reasonably efficient.



Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For 74 – 27 this involves partitioning the 27 into 20 and 7, and then subtracting from 74 the 20 and the 7 in turn. This use of partitioning is a useful step towards the most commonly used column method, decomposition

$$74 - 20 = 54$$

$$54 - 7 = 47$$

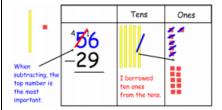
Expanded layout, leading to column method (Decomposition) Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where ones are placed under ones and tens under tens. This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills.

Expanded method 500 60 3

The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practicing the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.

Example: 56 - 29, **exchange** from the tens to the ones so that ones can be taken away using Dienes and place value counters.

Start by subtracting the ones, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying 'sixty take away forty', not 'six take away four'.



Introduce column subtraction with place value counters (Dienes could be used for those who need a less abstract representation)

For some children this will lead to exchanging, modelled using place value counters (or Dienes).

A number line and expanded column method may be compared next to each other.

The formal method should be seen as a more streamlined version of the expanded method, not a new method.

5 78 2

- 3 0 6

Ensure that children can explain the compact method, referring to the real value of the digits. They need to understand that they are repartitioning the 60 + 3 as 50 + 13.

Generalisation – identifying patterns

Noticing what happens to the digits when you count in tens and hundreds.

Odd – odd = even etc (see Year 2)

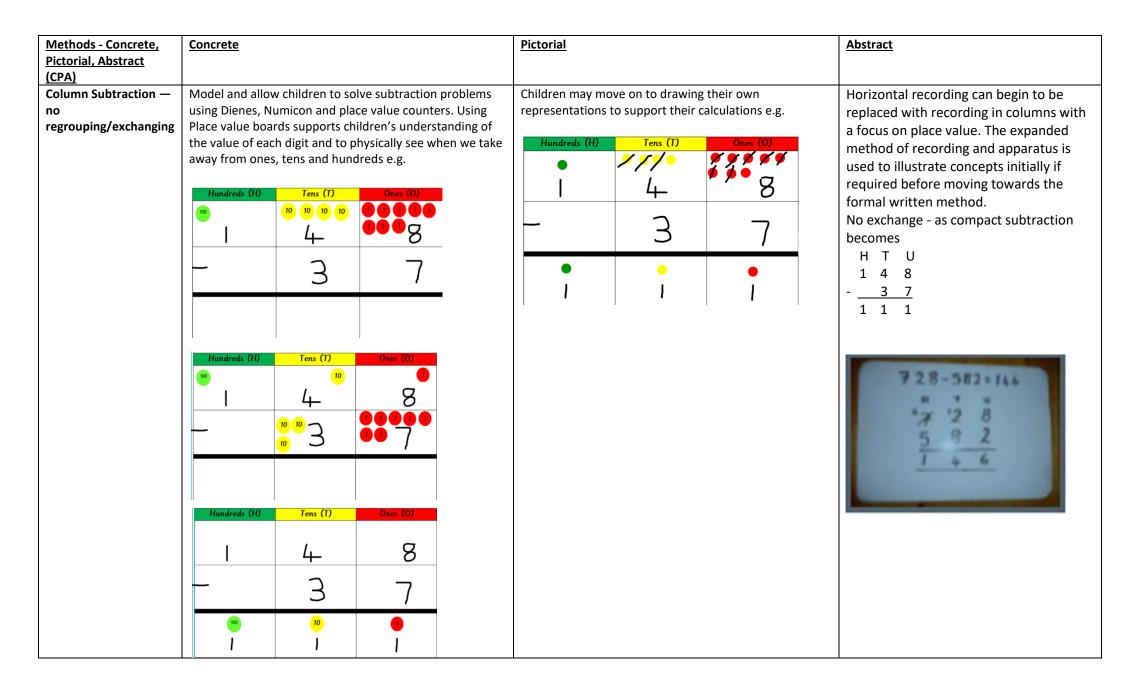
Inverses and related facts – develop fluency in finding related addition and subtraction facts.

Develop the knowledge that the inverse relationship can be used as a checking method.

Key Questions

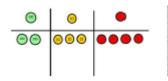
What do you notice? What patterns can you see?

When comparing two methods alongside each other: What's the same? What's different? Look at this number in the formal method; can you see where it is in the expanded method / on the number line.



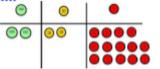
Column Subtraction with regrouping/exchanging

Some children prefer to use Base 10 materials (Dienes) to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

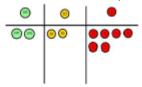


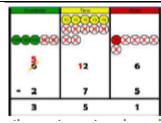
Make the larger number with the place value counters

Start with the ones, can I take away 8 from 4 easily? I need to exchange 1 of my tens for 10 ones.



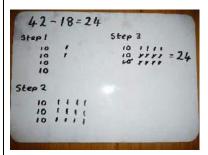
Now I can subtract my ones.





Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make. When confident, children can find their own way to record the exchange/regrouping.

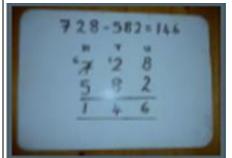
Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.





Children can start their formal written method by partitioning the number into clear place value columns.

Moving forward the children use a more compact method.



$$\frac{6}{7} - \frac{5}{7} = \frac{1}{7}$$

Year 4

Subtraction Learning Objectives

- subtract numbers with up to 4 digits using the formal written methods of columnar subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten
- solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number
- subtract fractions with the same denominator

Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate.

Children should continue to partition numbers in different ways.

They should be encouraged to choose from a range of strategies:

Counting forwards and backwards: 124 – 47, count back 40 from 124, then 4 to 80, then 3 to 77

Reordering: 28 + 75, 75 + 28 (thinking of 28 as 25 + 3)

Partitioning: counting on or back: 5.6 + 3.7, 5.6 + 3 + 0.7 = 8.6 + 0.7

Partitioning: bridging through multiples of 10: 6070 – 4987, 4987 + 13 + 1000 + 70

Partitioning: compensating -138 + 69, 138 + 70 - 1

Partitioning: using 'near' doubles - 160 + 170 is double 150, then add 10, then add 20, or double 160 and add 10, or double 170 and subtract 10

Partitioning: bridging through 60 to calculate a time interval – What was the time 33 minutes before 2.15pm?

Using known facts and place value to find related facts.

Mentally subtract values using an understanding of place value e.g. 10,000 - 9

Formal Written Calculations and Strategies

Missing number/digit problems: $456 + \Box = 710$;

 $1 \Box 7 + 6 \Box = 200$; $60 + 99 + \Box = 340$; $200 - 90 - 80 = \Box$; $225 - \Box = 150$; $\Box - 25 = 67$; $3450 - 1000 = \Box$; $\Box - 2000 = 900$

Mental methods

Should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

Formal Written Calculations and Strategies

Missing number/digit problems: $456 + \Box = 710$;

 $1 \Box 7 + 6 \Box = 200$; $60 + 99 + \Box = 340$; $200 - 90 - 80 = \Box$; $225 - \Box = 150$; $\Box - 25 = 67$; $3450 - 1000 = \Box$; $\Box - 2000 = 900$

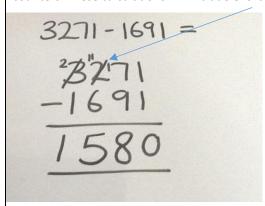
Mental methods

Should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

Written methods (progressing to 4-digits and introducing decimals)

Expanded column subtraction with decomposition, modelled with place value counters, progressing to calculations with 4-digit numbers.

If understanding of the expanded method with resources is secure, children will move on to the formal method of decomposition, which again can be initially modelled with place value counters. These calculations will include **exchanging** from the tens, hundreds and thousands (see below).



Children will subtract decimal numbers with the same number of decimal places with decomposition.

4	-63 - 2.91	=
	4:63	
-	2.91	
	1.72	
-		

Generalisation – identifying patterns

Investigate when re-ordering works as a strategy for subtraction. Eg. 20 - 3 - 10 = 20 - 10 - 3, but 3 - 20 - 10 would give a different answer.

Key Questions

What do you notice? What's the same? What's different? Can you convince me? How do you know?

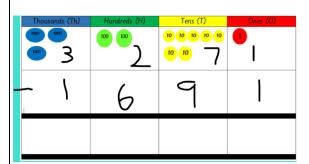
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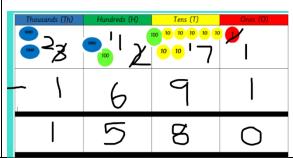
Methods - Concrete,	<u>Concrete</u>	<u>Pictorial</u>	<u>Abstract</u>
Pictorial, Abstract			
(CPA)			

Subtract with up to 4 digits.

Introduce decimal subtraction through context of money.

Children will solve subtraction problems using columnar subtraction where appropriate. To support their working and understanding place value boards and Dienes or place value counters will be used.





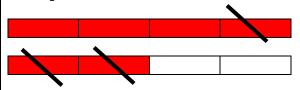
Children will draw their own representations or place value counters to support them solve subtraction problems using the columnar method.

Thousands (<u>Th</u>)	Hundreds (H)	Tens (T)	Ones (0)
28	ے ا'	רי	7
- 1	6	9	1
Ī	5	8	0

Children will subtract numbers with up to 4-digits using the formal written method of column subtraction with decomposition

Pupils continue to practise subtracting fractions with the same denominator to become fluent through a variety of increasingly complex problems beyond one whole.

Children can use manipulatives (such as counters, cubes or strips of paper) to support their understanding of improper fractions and subtracting fractions. They will also draw bar models to support them e.g.



$$\frac{6}{4} - \frac{3}{4} = \frac{3}{4}$$

Year 5

Subtraction Learning Objectives

- subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction)
- subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- subtract fractions with the same denominator and denominators that are multiples of the same number

Mental Strategies

Children should continue to count regularly, on and back, now including steps of powers of 10.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate.

Children should continue to partition numbers in different ways.

They should be encouraged to choose from a range of strategies:

Counting forwards and backwards in tenths and hundredths: 1.7 + 0.55

Reordering: 4.7 + 5.6 - 0.7, 4.7 - 0.7 + 5.6 = 4 + 5.6

Partitioning: counting on or back - 540 + 280, 540 + 200 + 80

Partitioning: bridging through multiples of 10:

Partitioning: compensating: 5.7 + 3.9, 5.7 + 4.0 - 0.1

Partitioning: using 'near' double: 2.5 + 2.6 is double 2.5 and add 0.1 or double 2.6 and subtract 0.1

Partitioning: bridging through 60 to calculate a time interval: It is 11.45. How many hours and minutes is it to 15.20?

Using known facts and place value to find related facts.

Mentally subtract values using an understanding of place value e.g. 5,000,000 - 20

Formal Written Calculations and Strategies

Missing number/digit problems: $6.45 = 6 + 0.4 + \Box$; $119 - \Box = 86$; $1000000 - \Box = 999000$; $600000 + \Box + 1000 = 671000$; $12462 - 2300 = \Box$

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

Written methods (progressing to more than 4-digits and decimals)

When understanding of the expanded method is secure, children will move on to the formal method of decomposition, which can be initially modelled with place value counters.

Progress to calculating with decimals, including those with different numbers of decimal places (see example in CPA approach).

	ž	"X	0'	'z	6	
_		2	1	2	8	
	2	8.	9	2	8	
			•			

Generalisation – identifying patterns

Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9.

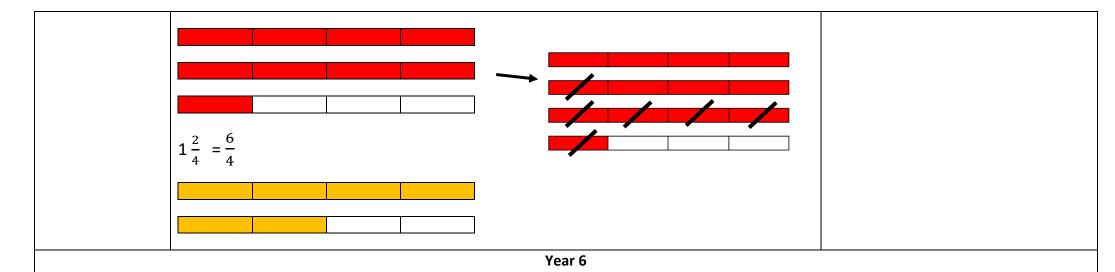
What do you notice about the differences between consecutive square numbers?

Investigate a - b = (a-1) - (b-1) represented visually.

Key Questions

What do you notice? What's the same? What's different? Can you convince me? How do you know?

	Year 5				
Methods - Concrete, Pictorial, Abstract (CPA)	Methods - Concrete, Pictorial, Abstract (CPA)	Methods - Concrete, Pictorial, Abstract (CPA)	Methods - Concrete, Pictorial, Abstract (CPA)		
Subtract with at least 4 digits, including money and measures. Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal.	Place value grids and counters or Dienes may be used to support subtraction. Decimal counters may be used to support when subtracting decimal numbers e.g. Tens of thousands Thousands Hundreds Tens Ones tenths Hundredths Thousandths - 1 4 2 5	Place value grids will be used to support children when subtracting decimals e.g.	Formal written method. This will lead to an understanding of subtracting any number including decimals. 5 6 10 4 5 11 - 1 4 2 • 5		
	Children will use their knowledge of exchanging from previous year groups.	They may still use counters to support this.	4 6 2 • 6		
Practise subtracting fractions where calculations exceed one as a mixed number.	Children will learn that they must always convert mixed n concrete objects Cuisenaire may be used to visualise. Show me Show me the Show me half of red. other half of red. halves of the convert to $3\frac{1}{4} - 1\frac{2}{4} = 3\frac{1}{4} = \frac{13}{4}$	Children will learn that they must always convert mixed numbers to improper fractions when subtracting. $3\frac{1}{4} - 1\frac{2}{4} = \frac{13}{4} - \frac{6}{4}$ $\frac{13}{4} - \frac{6}{4} = \frac{7}{4} = 1\frac{3}{4}$			



- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Revisit and review previous year group objectives
- subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions

Mental Strategies

Consolidate previous years.

Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. $20 - 5 \times 3 = 5$; $(20 - 5) \times 3 = 45$

Children will be reminded to use BODMAS (see multiplication calculation policy) to ensure correct order of operations if necessary.

Formal Written Calculations and Strategies

Missing number/digit problems: \Box and # each stand for a different number. # = 34. # + # = \Box + #. What is the value of \Box ? What if # = 28? What if # = 21 10 000 000 = 9 000 100 + \Box

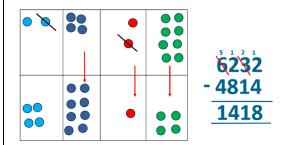
 $7 - 2 \times 3 = \Box$; $(7 - 2) \times 3 = \Box$; $(\Box - 2) \times 3 = 15$

Find pairs of missing numbers e.g. 2a + b = 20 a = 10 b = 0

A = 5 b = 10 etc.

<u>Mental methods</u> should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. <u>Written methods (progressing to more than 4-digits and decimals)</u>

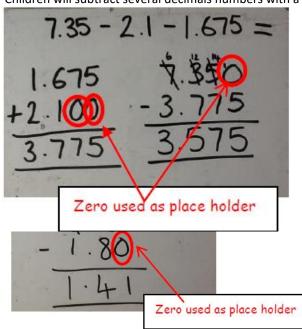
As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with decomposition to be secured.

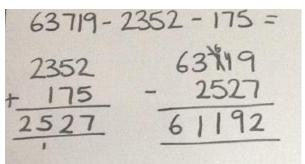


Children will subtract several numbers of increasing complexity and be taught to combine some of the numbers so that the subtraction can be completed.

Children will subtract decimal numbers with a different number of decimal places with decomposition

Children will subtract several decimals numbers with a different number of decimal places be taught to combine some of the numbers so that the subtraction can be completed.





<u>Generalisation – identifying patterns</u>

Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as BODMAS, or could be encouraged to design their own ways of remembering.

Sometimes, always or never true? Subtracting numbers makes them smaller.

Key Questions

What do you notice?

What's the same? What's different?

Can you convince me?

How do you know? Prove it.			
Methods - Concrete, Pictorial, Abstract (CPA)	Concrete	<u>Pictorial</u>	Abstract
Subtract with increasingly large and more complex numbers and decimal values.	As previous year groups.	As previous year groups.	Formal written method. This will lead to an understanding of subtracting any number including decimals. Zero is used as the place holder.
Subtract fractions and mixed numbers with different denominators using the concept of equivalent fractions.	Children May use Cuisenaire to find equivalents fractions e.g.	Fraction walls may be used to support children to find common denominators and equivalent fractions.	$\frac{4}{6} - \frac{1}{3}$ find a common denominator = $\frac{1}{3} = \frac{2}{6} \text{ (x by 2)}$ $\frac{4}{6} - \frac{2}{6} = \frac{2}{6}$