Katherine Semar Division Calculation Policy

Progression through Calculations for Division

Division

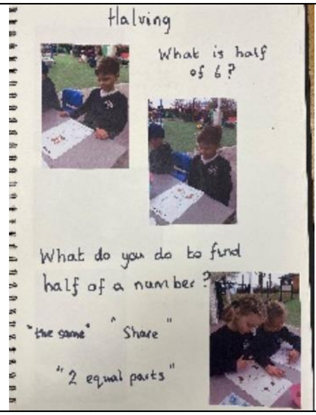
EYFS

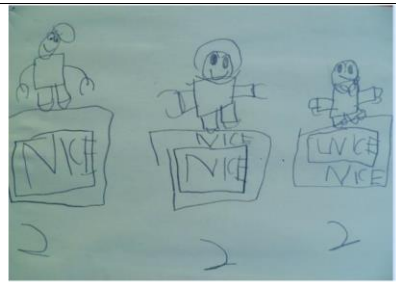
Before division can be introduced, children should have a secure knowledge of number facts and subtraction. Children will be introduced to the concept of halving and sharing orally and through practical games and activities.
Pupils should have many practical experiences of sharing objects e.g. sharing between two people or finding half of a group of objects.

Use a range of concrete materials to show a number and then share equally. Then move onto pictorial representations.

Links to ELG:

Automatically recall number bonds up to five and some number bonds to ten including double facts.
Explore and represent patterns within numbers up to ten, including even and odds, double facts and how quantities can be distributed equally

Methods - Concrete, Pictorial, Abstract (CPA)	**Concrete**	**Pictorial**	**Abstract**
Halving	 Finding half of eight		

Sharing and Grouping	 Making equal groups of objects  We have eight mice that are shared equally between two owls. Each owl has four mine.	 Sharing six biscuits equally between three people	

Year 1

Division Learning Objectives

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher

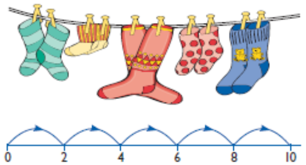
Guidance (non-statutory)

- Through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; and finding simple fractions of objects, numbers and quantities.
- They make connections between arrays, number patterns, and counting in 2s, 5s and 10s.

Mental Strategies

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.

They should begin to recognise the number of groups counted to support understanding of relationship between multiplication and division.



$2+2+2+2+2=10$
$2\times5=10$
2 multiplied by 5
5 pairs
5 hops of 2

Children should begin to understand division as both sharing and grouping.

Sharing – 6 sweets are shared between 2 people. How many do they have each?

Grouping-
How many 2's are in 6?

They should use objects to group and share amounts to develop understanding of division in a practical sense.
E.g. using Numicon to find out how many 5's are in 30? How many pairs of gloves if you have 12 gloves?

Children should begin to explore finding simple fractions of objects, numbers and quantities.

E.g.*16 children went to the park at the weekend. Half that number went swimming. How many children went swimming?*

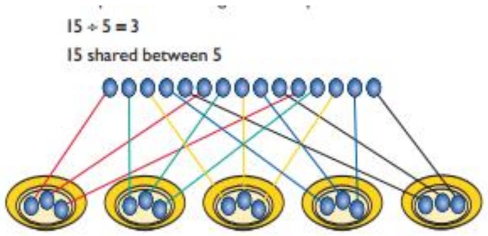
Formal Written Calculations and Strategies

Children must have secure counting skills- being able to confidently count in 2s, 5s and 10s.
Children should be given opportunities to reason about what they notice in number patterns.

Group AND **share small quantities- understanding the difference between the two concepts.**

Sharing

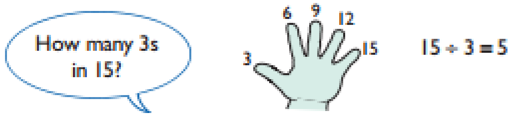
Develops importance of one-to-one correspondence.



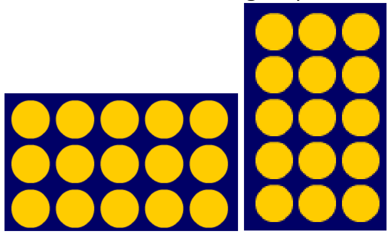
Children should be taught to share using concrete apparatus.

<u>Grouping</u>

Children should apply their counting skills to develop some understanding of grouping.



Use of arrays as a pictorial representation for division. 15 ÷ 3 = 5 There are 5 groups of 3.
15 ÷ 5 = 3 There are 3 groups of 5.



Children should be able to find ½ and ¼ and simple fractions of objects, numbers and quantities.

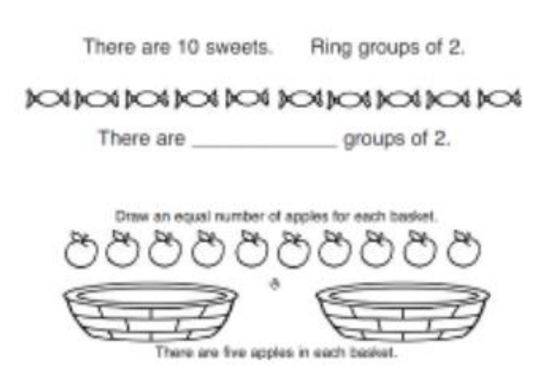
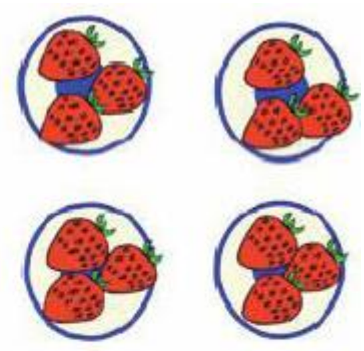
<u>Generalisation – identifying patterns</u>

True or false? I can only halve even numbers.

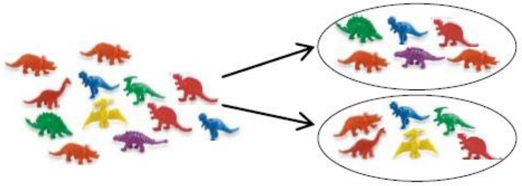
Grouping and sharing are different types of problems. Some problems need solving by grouping and some by sharing. Encourage children to practically work out which they are doing.

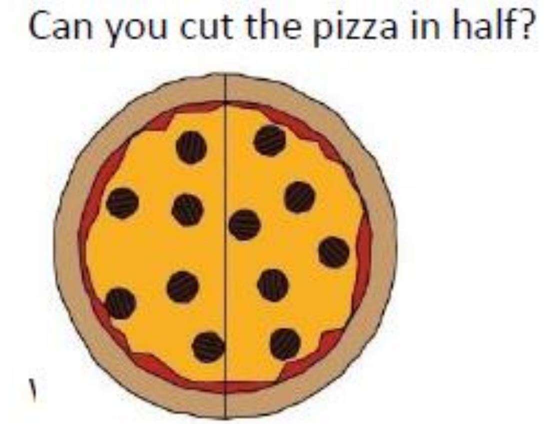
<u>Key Questions</u>

How many groups of…?
How many in each group?
Share… equally into…
What can do you notice?

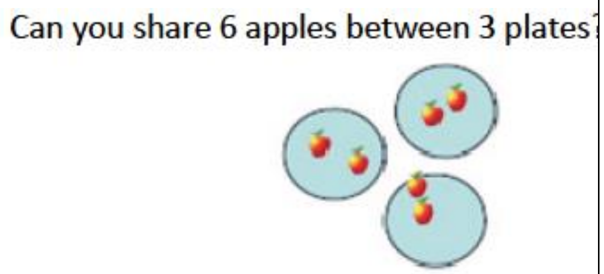
Methods - Concrete, Pictorial, Abstract (CPA)	**Concrete**	**Pictorial**	**Abstract**
Sharing objects into groups	 Holly and David shared ten sweets equally into two bowls. How many did they get each? The emphasis is made on **equal** groups/amounts. 	 Children will understand equal groups and share items out in play scenarios. Share 12 cakes between 3 people equally: 	The division symbol is not formally taught at this stage. The focus is on sharing objects into equal groups and halving. Children experience early division by sharing objects and counting how many in each group. 

Children will be taught to associate 'half' with dividing by two and recognise, find and name a half as one of two equal parts





Children will be given a word problem to complete either practically or using pictorial representations.



Year 2

Division Learning Objectives

- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs
- Show that multiplication of 2 numbers can be done in any order (commutative) and division of 1 number by another cannot
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts

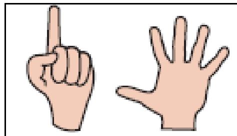
Guidance (non-statutory)

- Pupils use a variety of language to describe multiplication and division.
- Pupils are introduced to the multiplication tables. They practise to become fluent in the 2, 5 and 10 multiplication tables and connect them to each other. They connect the 10 multiplication table to place value, and the 5 multiplication table to the divisions on the clock face. They begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations.
- Pupils work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition. They begin to relate these to fractions and measures (for example, 40 ÷ 2 = 20, 20 is a half of 40). They use commutativity and inverse relations to develop multiplicative reasoning (for example, 4 × 5 = 20 and 20 ÷ 5 = 4).

Mental Strategies

Children should count regularly, on and back, in steps of 2, 3, 5 and 10.

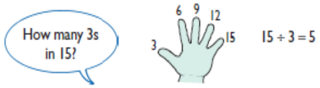
Children who are able to count in twos, threes, fives and tens can use this knowledge to work out other facts such as 2 × 6, 5 × 4, 10 × 9. Show the children how to hold out their fingers and count, touching each finger in turn. So for 2 × 6 (six twos), hold up 6 fingers:



Touching the fingers in turn is a means of keeping track of how far the children have gone in creating a sequence of numbers. The physical action can later be visualised without any actual movement.

This can then be used to support finding out 'How many 3's are in 18?' and children count along fingers in 3's therefore making link between multiplication and division.

Children should continue to develop understanding of division as sharing **and** grouping.



15 pencils shared between 3 pots, how many in each pot?

Children should be given opportunities to find a half, a quarter and a third of shapes, objects, numbers and quantities. Finding a fraction of a number of objects to be related to sharing.

They will explore visually and understand how some fractions are equivalent – e.g. two quarters is the same as one half.

Use children's intuition to support understanding of fractions as an answer to a sharing problem.

If 1 chocolate bar is shared equally between 4 people, what fraction would they each get? $1 \div 4 = \frac{1}{4}$

Formal Written Calculations and Strategies

÷ = signs and missing numbers

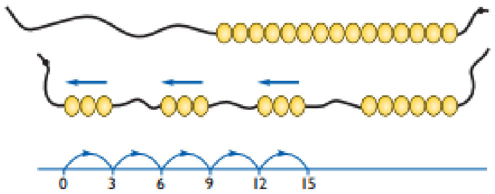
$6 \div 2 = \square$	$\square = 6 \div 2$
$6 \div \square = 3$	$3 = 6 \div \square$
$\square \div 2 = 3$	$3 = \square \div 2$
$\square \div \nabla = 3$	$3 = \square \div \nabla$

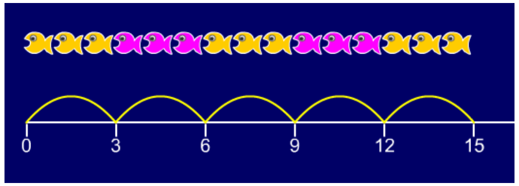
Know and understand sharing and grouping- introducing children to the ÷ sign.
Children should continue to use grouping and sharing for division using practical apparatus, arrays and pictorial representations.

Grouping using a number line

Group from zero in jumps of the divisor to find our 'how many groups of 3 are there in 15?'

15 ÷ 3 = 5





Continue work on arrays. Support children to understand how multiplication and division are inverse. Look at an array – what do you see?

<u>Generalisation – identifying patterns</u>

Noticing how counting in multiples if 2, 5 and 10 relates to the number of groups you have counted (introducing times tables)

An understanding of the more you share between, the less each person will get (e.g. would you prefer to share these grapes between 2 people or 3 people? Why?)

Secure understanding of grouping means you count the number of groups you have made. Whereas sharing means you count the number of objects in each group.

<u>Key Questions</u>

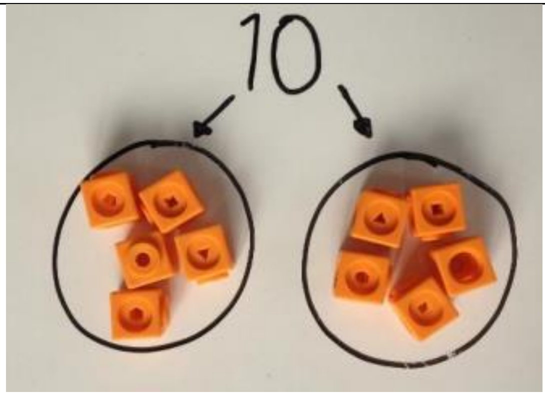
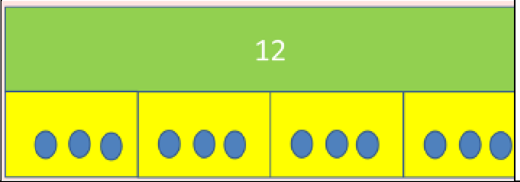
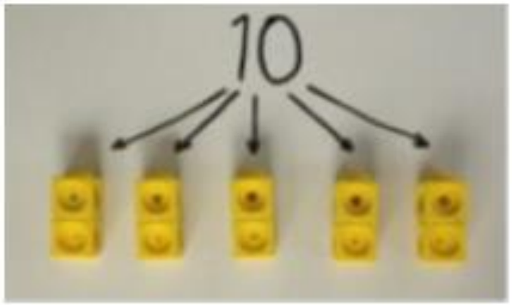
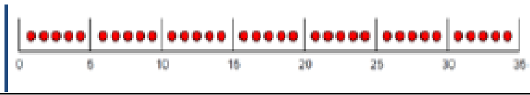
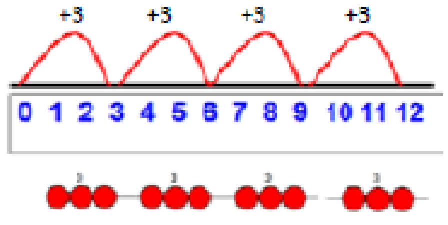
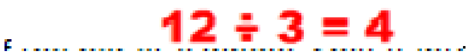
How many 10s can you subtract from 60?

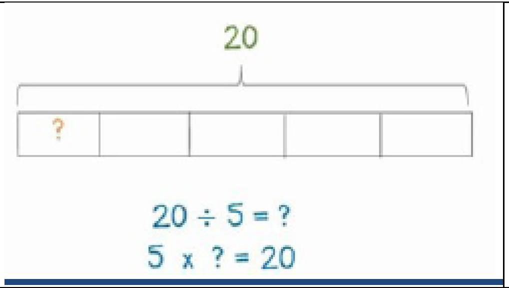
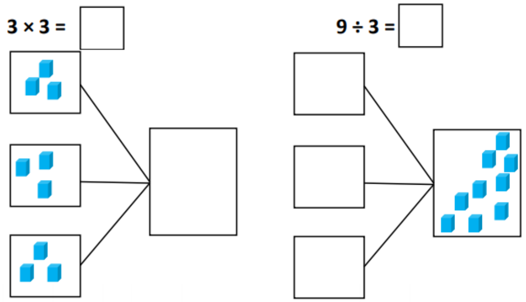
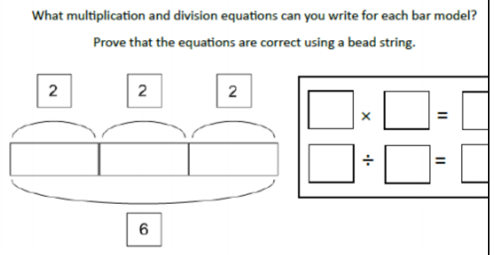
I think of a number and double it. My answer is 8. What was my number?

If 12 x 2 = 24, what is 24 ÷ 2?

Questions in the context of money and measures (e.g. how many 10p coins do I need to have 60p? How many 100ml cups will I need to reach 600ml?)

<u>Methods - Concrete, Pictorial, Abstract (CPA)</u>	**<u>Concrete</u>**	**<u>Pictorial</u>**	**<u>Abstract</u>**
Division as sharing		Children use pictures or shapes to share quantities.  8 ÷ 2 = 4	10 ÷ 2 = 5

		 Children use bar modelling to show and support understanding. $12 \div 4 = 3$ 	
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping   Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.	$28 \div 7 = 4$ Divide 28 into 7 groups. How many are in each group?

			
Use of part-part-whole model to establish the inverse relationship between multiplication and division			$2 \times 6 = 12$ $6 \times 2 = 12$ $12 \div 2 = 6$ $12 \div 6 = 2$

Year 3

<u>Division Learning Objectives</u>

- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects

Guidance (non-statutory)

- Pupils continue to practise their mental recall of multiplication tables when they are calculating mathematical statements in order to improve fluency. Through doubling, they connect the 2, 4 and 8 multiplication tables.
- Pupils develop efficient mental methods, for example, using commutativity and associativity (for example, 4 × 12 × 5 = 4 × 5 × 12 = 20 × 12 = 240) and multiplication and division facts (for example, using 3 × 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (30 × 2 = 60, 60 ÷ 3 = 20 and 20 = 60 ÷ 3).

- Pupils develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of short multiplication and division.
- Pupils solve simple problems in contexts, deciding which of the 4 operations to use and why. These include measuring and scaling contexts, (for example 4 times as high, 8 times as long etc) and correspondence problems in which m objects are connected to n objects (for example, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).

<u>Mental Strategies</u>

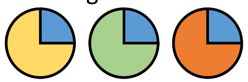
Children should count regularly, on and back, in steps of 3, 4 and 8. Children are encouraged to use what they know about known times table facts to work out other times tables.
This then helps them to make new connections (e.g. through doubling they make connections between the 2, 4 and 8 times tables).

Children will make use multiplication and division facts they know to make links with other facts.
3 x 2 = 6, 6 ÷ 3 = 2, 2 = 6 ÷ 3
30 x 2 = 60, 60 ÷ 3 = 20, 2 = 60 ÷ 30

They should be given opportunities to solve grouping and sharing problems practically (including where there is a remainder but the answer needs to given as a whole number)
e.g. Pencils are sold in packs of 10. How many packs will I need to buy for 24 children?

Children should be given the opportunity to further develop understanding of division (sharing) to be used to find a fraction of a quantity or measure.

Use children's intuition to support understanding of fractions as an answer to a sharing problem.
3 apples shared between 4 people = $\frac{3}{4}$



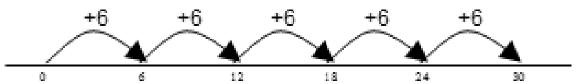
<u>Formal Written Calculations and Strategies</u>

÷ = signs and missing numbers

Continue using a range of equations as in year 2 but with appropriate numbers.

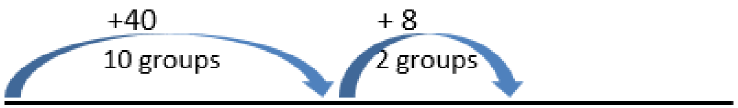
Grouping

How many 6's are in 30?
30 ÷ 6 can be modelled as:



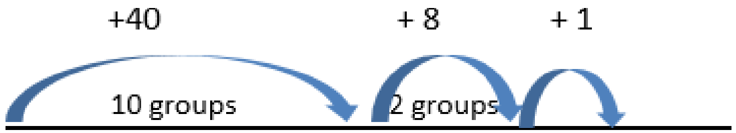
Becoming more efficient using a number line

Children need to be able to partition the dividend in different ways.
48 ÷ **4 = 12**



Remainders

49 ÷ 4 = 12 r1



Sharing – 49 shared between 4. How many left over?
Grouping – How many 4s make 49. How many are left over?
Place value counters can be used to support children apply their knowledge of grouping.

For example:
60 ÷ 10 = How many groups of 10 in 60?
600 ÷ 100 = How many groups of 100 in 600?

Generalisation – identifying patterns

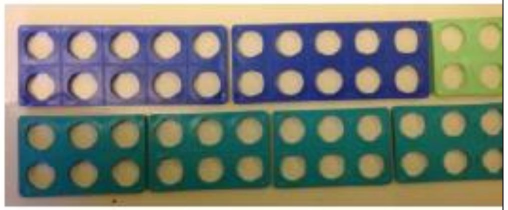
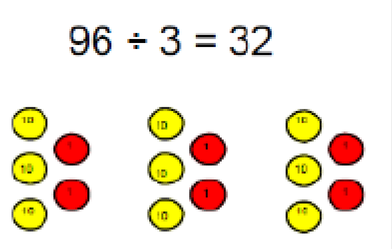
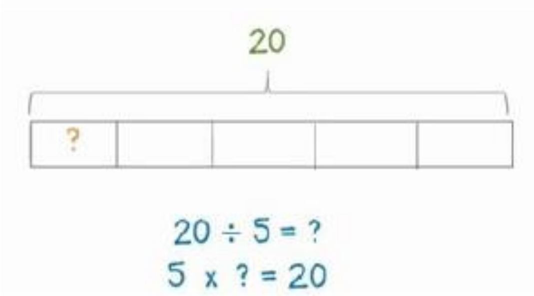
Inverses and related facts – develop fluency in finding related multiplication and division facts.
Develop the knowledge that the inverse relationship can be used as a checking method.

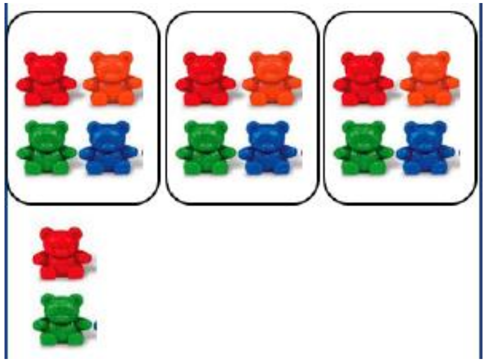
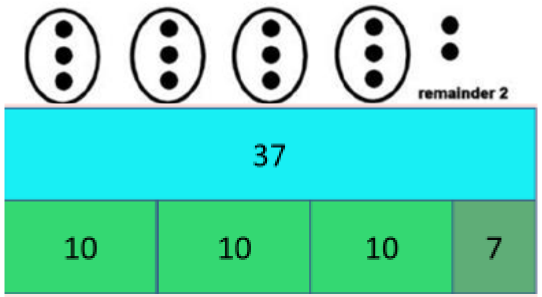
Key Questions

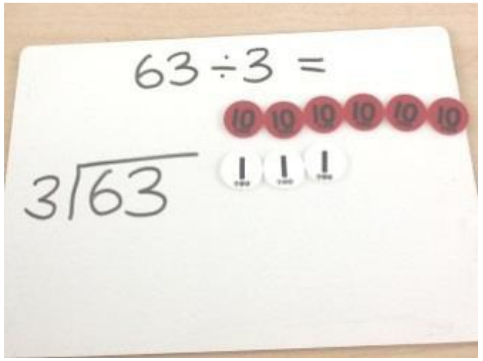
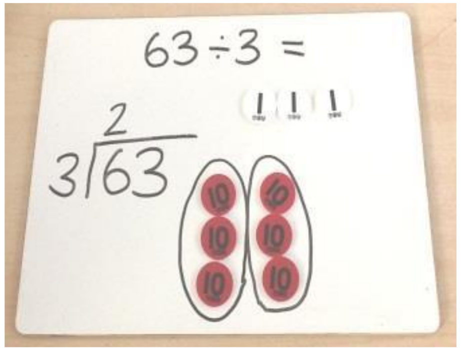
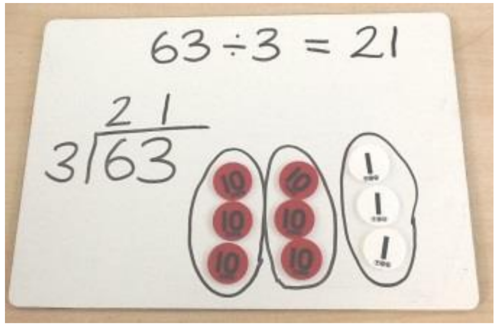
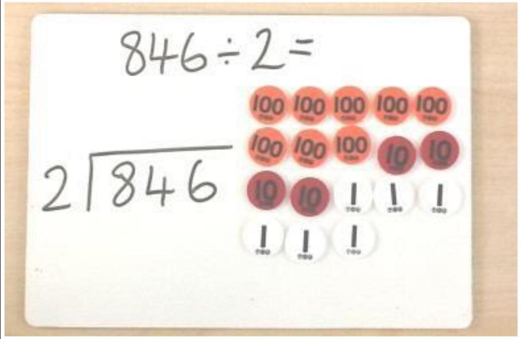
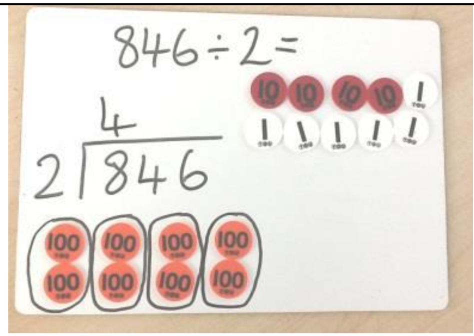
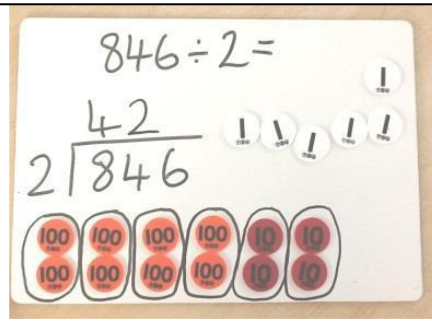
Questions in the context of money and measures that involve remainders (e.g. How many lengths of 10cm can I cut from 81cm of string? You have £54. How many £10 teddies can you buy?)

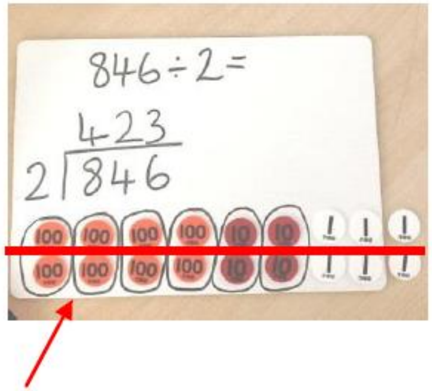
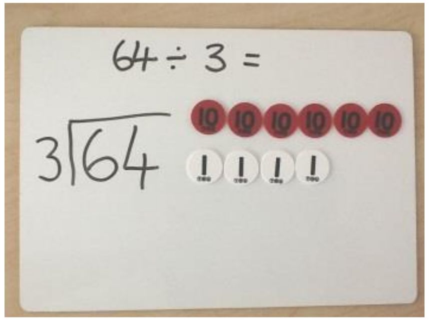
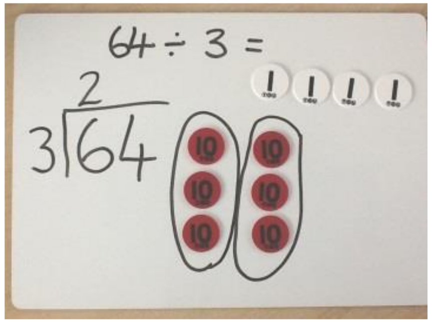
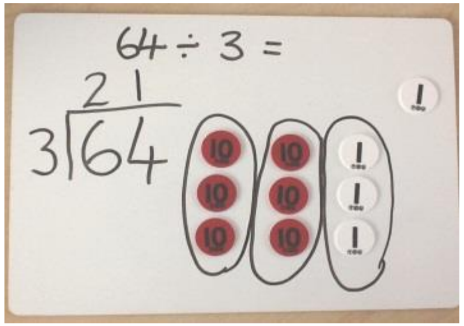
What is the missing number? 17 = 5 x 3 + __
__ = 2 x 8 + 1

Methods - Concrete, Pictorial, Abstract (CPA)	**Concrete**	**Pictorial**	**Abstract**

Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 	Continue to use bar modelling to aid solving division problems. 	How many groups of 6 in 24? 24 ÷ 6 = 4
	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. E.g. 48 ÷ 6 = 5 6 x 8 = 48 48 ÷ 8 = 6 8 x 6 = 48	Draw an array and use lines to split the array into groups to make multiplication and division sentences. 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4

			
Division with remainders.	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders.	Complete written divisions and show the remainder using r. 
Short division (dividing a 2digit number	Create the dividend using Place Value counters.	Group the tens counters according to the divisor and write the number of groups above the line in the tens column.	Group the tens counters according to the divisor and write the number of groups above the line in the tens column. The quotient can be seen across the groups.

by a single digit number) Children will use practical resources to support the short division method and will be encouraged to use multiples of the divisor to assist (TO ÷ O)	  
Short division (dividing a 3digit number by a single digit number) Children will use practical resources to support the short division method and will be encouraged to use multiples of the divisor to assist (HTO ÷ O)	Create the dividend using Place Value counters.  Group the 100s counters according to the divisor. Write the number of groups above the line in the hundreds column.  Next, group the 10s counters according to the divisor. Write the number of groups above the line in the tens column.  Next, group the 1s counters according to the divisor. Write the number of groups above the line in the 1s column.

	 The quotient can be seen across the groups.
Short division – 2digit numbers divided by single digit numbers with remainders (TO ÷ O r. O)	Create the dividend using Place Value counters.  Starting with tens counters, group them according to the divisor. Write the number of groups in the tens column above the line.  Next, group the ones according to the divisor and arrange next to the groups of ten. Write the number of groups above the line in the ones column.  Any counters that cannot be grouped are the remainder. Write this at the end as 'r1'.

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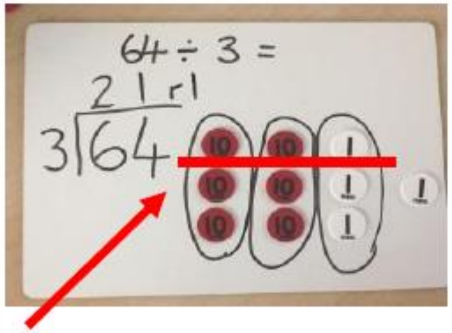
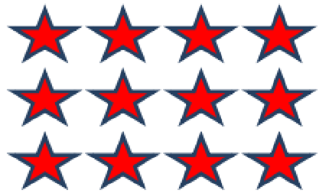
<td>

As you look across each group, the quotient can be seen.</td>
</tr>
<tr>
<td>Fractions of amounts and numbers (using division).</td>
<td>Concrete

$\frac{1}{10}$

Pictorial
There are 12 <u>stars</u>. $\frac{1}{3}$ of the <u>stars</u> is equal to <u>4 stars</u>

Children use the same sentence stem to express other relationships. For example:
There are 12 <u>stars</u>. $\frac{1}{4}$ of the <u>stars</u> is equal to <u>3 stars</u>
There are 12 <u>stars</u>. $\frac{1}{2}$ of the <u>stars</u> is equal to <u>6 stars</u></td>
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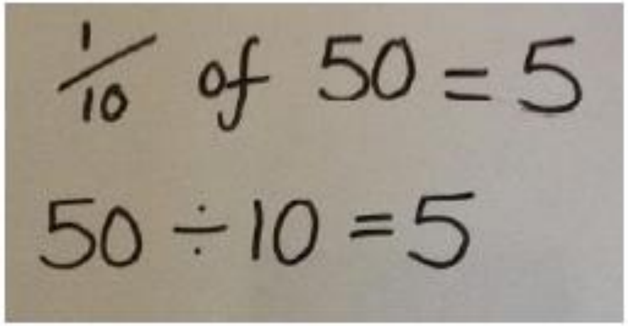
</table>

Abstract



Year 4

<u>Division Learning Objectives</u>

- recall multiplication and division facts for multiplication tables up to 12 × 12
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

Guidance (non-statutory)

- Pupils continue to practise recalling and using multiplication tables and related division facts to aid fluency.
- Pupils practise mental methods and extend this to 3-digit numbers to derive facts, (for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6).
- Pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers
- Pupils write statements about the equality of expressions (for example, use the distributive law 39 × 7 = 30 × 7 + 9 × 7 and associative law (2 × 3) × 4 = 2 × (3 × 4)). They combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations for example, 2 x 6 x 5 = 10 x 6 = 60.
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as the numbers of choices of a meal on a menu, or 3 cakes shared equally between 10 children.

<u>Mental Strategies</u>

<u>÷ = signs and missing numbers</u>

Continue using a range of equations as in year 3 but with appropriate numbers.

<u>Sharing, Grouping and using a number line</u>

Children will continue to explore division as sharing and grouping, and to represent calculations on a number line until they have a secure understanding. Children should progress in their use of written division calculations:

- Using tables facts with which they are fluent
- Experiencing a logical progression in the numbers they use, for example:

1. Dividend just over 10x the divisor, e.g. 84 ÷ 7
2. Dividend just over 10x the divisor when the divisor is a teen number, e.g. 173 ÷ 15 (learning sensible strategies for calculations such as 102 ÷ 17)
3. Dividend over 100x the divisor, e.g. 840 ÷ 7
4. Dividend over 20x the divisor, e.g. 168 ÷ 7

All of the above stages should include calculations with remainders as well as without. Remainders should be interpreted according to the context. (i.e. rounded up or down to relate to the answer to the problem)

Children will be encouraged to use their known multiplication and division facts to support them solve other calculations.

For example:

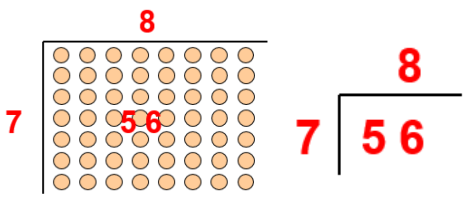
840 ÷ 7 = 120

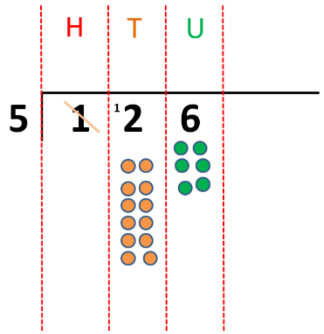
I know that 7 x 12 = 84
Therefore 7 x 120 = 840

I know that 7 x **100** = 700
And 7 x **20** = 140
100 + 20 = 120

__Formal Written Calculations and Strategies__

Alongside pictorial representations and the use of models and images, children should progress onto short division using a bus stop method.
Place value counters can be used to support children apply their knowledge of grouping. Reference should be made to the value of each digit in the dividend.
Formal short division should only be introduced once children have a good understanding of division, its links with multiplication and the idea of 'chunking up' to find a target number (see use of number lines above)
Short division to be modelled for understanding using place value counters as shown below. Calculations with 2 and 3-digit dividends.





Each digit as a multiple of the divisor
'How many groups of 3 are there in the hundreds column?'
'How many groups of 3 are there in the tens column?'
'How many groups of 3 are there in the units/ones column?'

Generalisation – identifying patterns
True or false? Dividing by 10 is the same as dividing by 2 and then dividing by 5. Can you find any more rules like this?
Is it sometimes, always or never true that □ ÷ Δ = Δ ÷ □?

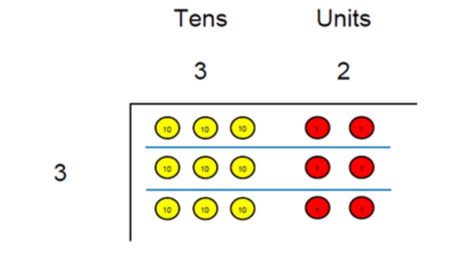
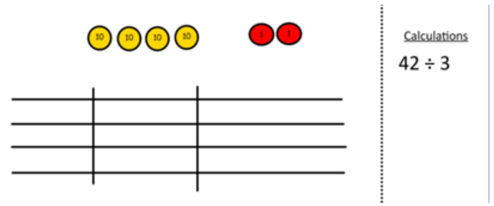
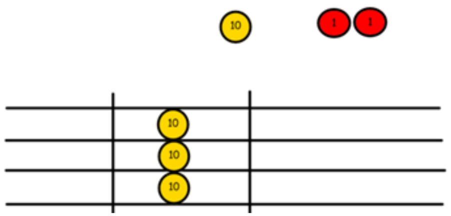
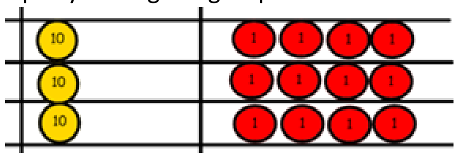
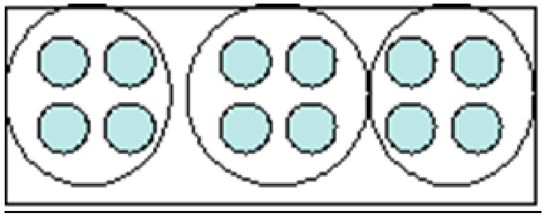
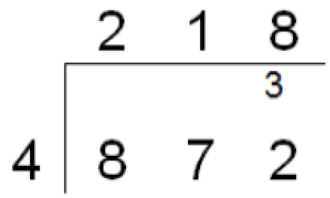
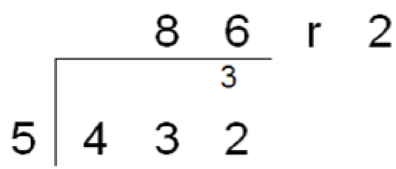
Inverses and deriving facts. 'Know one, get lots free!' e.g.: 2 x 3 = 6, so 3 x 2 = 6, 6 ÷ 2 = 3, 60 ÷ 20 = 3, 600 ÷ 3 = 200 etc.

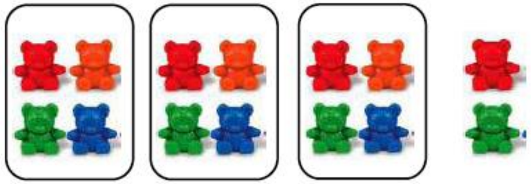
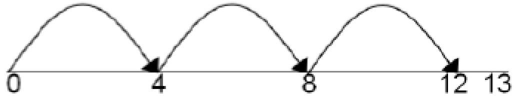
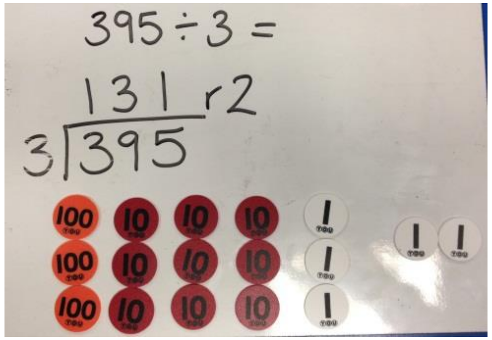
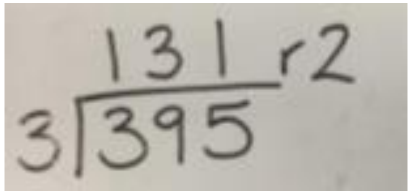
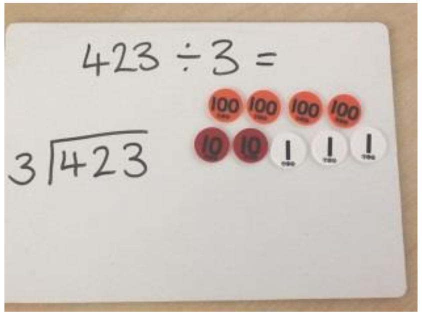
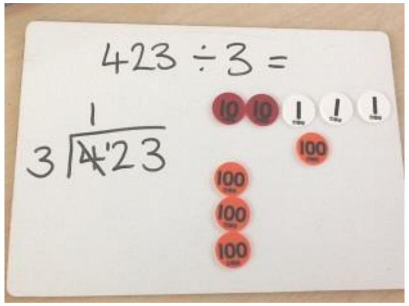
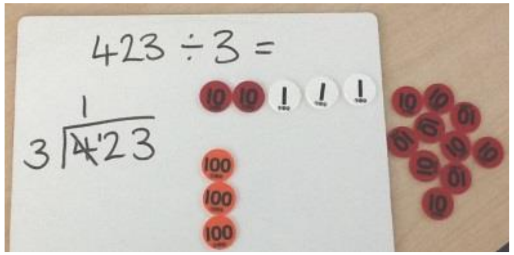
Sometimes, always, never true questions about multiples and divisibility. (When looking at the examples on this page, remember that they **may not** be 'always true'!) E.g.:

- Multiples of 5 end in 0 or 5.
- The digital root of a multiple of 3 will be 3, 6 or 9.
- The sum of 4 even numbers is divisible by 4.

Key Questions
What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?
What is the inverse?
How can you use your multiplication and division facts to support you?

Methods - Concrete, Pictorial, Abstract (CPA)	**Concrete**	**Pictorial**	**Abstract**

Short division. Divide at least 3 digit numbers by 1 digit.	 Use place value counters to divide using the formal written method  42 ÷ 3= Start with the greatest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  Exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Children can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  They will then be encouraged to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder. 
Division with a remainder	Divide objects between groups and see Many are left over. 14 ÷ 3 =	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.	

		 Draw dots and group them to divide an amount and clearly show a remainder.	
Short division with a remainder		Children draw pictorial representations to support their division. They may use place value counters initially and then draw the counters moving forward.	Children will use their division facts to support them divide 3 and 4 digit numbers by a single digit number. 395 ÷ 3 = 131 r. 2 
Use practical resources to support the short division method where exchange across place value columns occurs. (HTO ÷ O)	Create the dividend using Place Value counters.  Next, group the 10s counters according to the divisor and write the number of groups above the line in the tens column.	Group the hundreds counters according to the divisor. Write the number of groups above the line in the hundreds column.  Group the ones counters according to the divisor and write the number of groups above the line in the ones column.	Exchange the left over 100s counter for ten 10s counters and represent this beneath the line in the tens column. 

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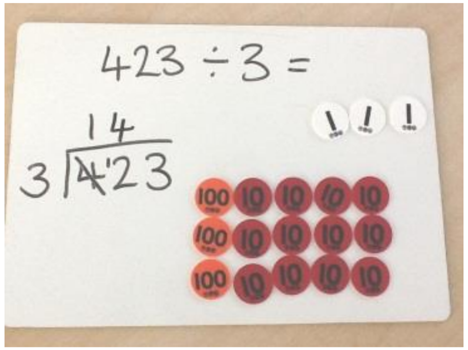
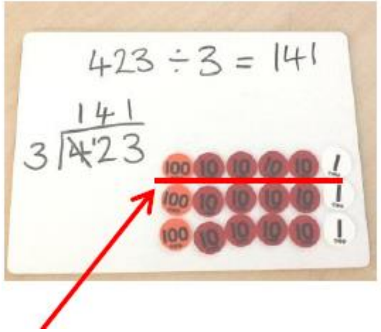




The quotient can be seen across each group

Find the effect of dividing a 1 or 2-digit number by 10 and 100; identifying the value of the digits in the answer as units, tenths and hundredths

Children must have a secure understanding that the decimal point does not move. It is the digits that move place.

Dividing by 10
When you divide a decimal number by 10 you move all the digits **one place to the right**. The number becomes 10 times **smaller**.

Example: 3,502 ÷ 10 = 350.2
You can see that the digits move along to the right. Thousands move to hundreds, and the others follow like this:

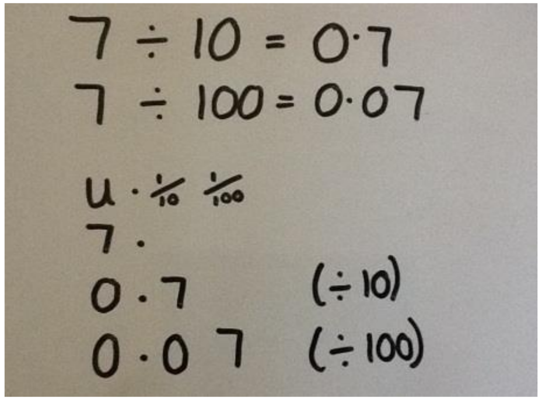
Th	H	T	U		$\frac{1}{10}$	$\frac{1}{100}$
3	5	0	2			
	3	5	0	.	2	

Th = thousands

H = hundreds

T= tens

U = units



Dividing by 100
When you divide a decimal number by 100 you move all the digits **two places to the right**. The number becomes **100 times smaller**.

Example: 3,502 ÷ 100 = 35.02

Th	H	T	U		$\frac{1}{10}$	$\frac{1}{100}$
3	5	0	2			
		3	5	.	0	2

Th = thousands

H = hundreds

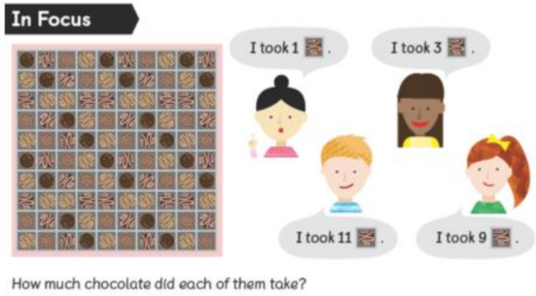
T= tens

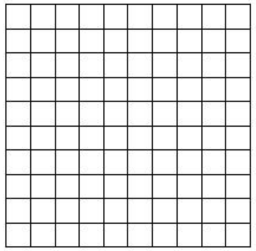
U = units

Children will begin by using place value mats and counters/number cards to support dividing 1 digit numbers by 10 and 100.

Count up and down in hundredths; recognise that hundredths arise when dividing an object by a hundred and dividing tenths by ten

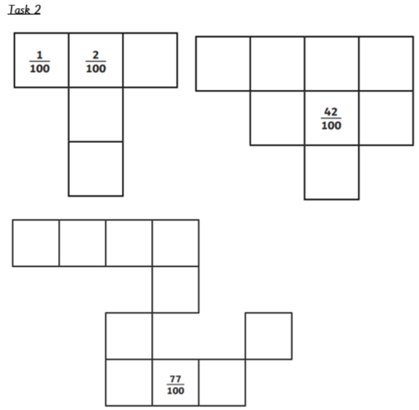
Concrete and Pictorial





Concrete objects such as Dienes will be used to represent hundredths.
The children then may use a grid to support them (pictorial)

Abstract



Year 5

<u>Division Learning Objectives</u>

- identify multiples and factors, including finding all factor pairs of a number, and common factors of 2 numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally, drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000
- recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})
- solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates

Guidance (non-statutory)

- Pupils practise and extend their use of the formal written methods of short multiplication and short division. They apply all the multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations.
- They use and understand the terms factor, multiple and prime, square and cube numbers.

- Pupils interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (for example, $98 \div 4 = \frac{98}{4} = 24 \text{ r } 2 = 24\frac{1}{2} = 24.5 \approx 25$).
- Pupils use multiplication and division as inverses to support the introduction of ratio in year 6, for example, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1,000 in converting between units such as kilometres and metres
- Distributivity can be expressed as $a(b + c) = ab + ac$.
- They understand the terms factor, multiple and prime, square and cube numbers and use them to construct equivalence statements (for example, 4 x 35 = 2 x 2 x 35; 3 x 270 = 3 x 3 x 9 x 10 = 9^2 x 10).

  Pupils use and explain the equals sign to indicate equivalence, including in missing number problems (for example 13 + 24 = 12 + 25; 33 = 5 x ?).

Mental Strategies

÷ = signs and missing numbers

Continue using a range of equations but with appropriate numbers

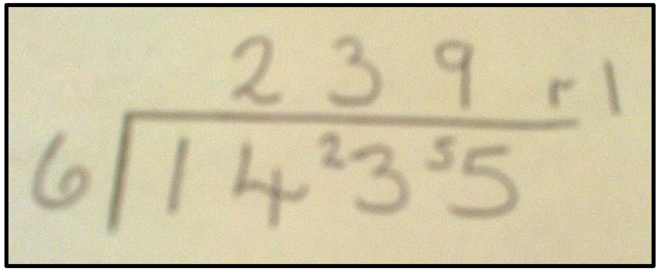
Sharing and Grouping and using a number line

Children will continue to explore division as sharing and grouping, and to represent calculations on a number line as appropriate.
Quotients should be expressed as decimals and fractions

Formal Written Calculations and Strategies

Continued as shown in Year 4, leading to the efficient use of a formal method. The language of grouping to be used.

E.g. 1435 ÷ 6



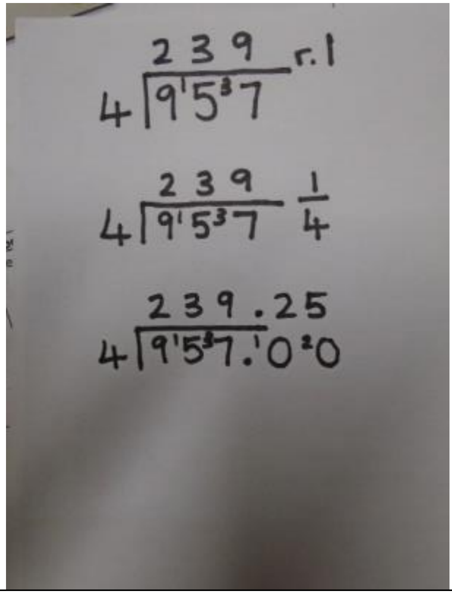
Children begin to practically develop their understanding of how express the remainder as a decimal or a fraction. Ensure practical understanding allows children to work through this (e.g. what could I do with this remaining 1? How could I share this between 6 as well?)

Formal Written Calculations and Strategies

Continued as shown in Year 4, leading to the efficient use of a formal method. The language of grouping to be used.

Children begin to practically develop their understanding of how express the remainder as a decimal or a fraction. Ensure practical understanding allows children to work through this (e.g. what could I do with this remaining 1? How could I share this between 6 as well?)

E.g. 957 ÷ 3 =



<u>Generalisation – identifying patterns</u>

The = sign means equality. Take it in turn to change one side of this equation, using multiplication and division, e.g.

Start: **24 = 24**

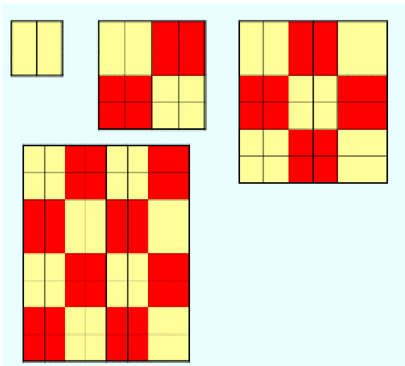
Player 1: **4 x 6 = 24**

Player 2: **4 x 6 = 12 x 2**

Player 1: **48 ÷ 2 = 12 x 2**

Sometimes, always, never true questions about multiples and divisibility:

- If the last two digits of a number are divisible by 4, the number will be divisible by 4.
- If the digital root of a number is 9, the number will be divisible by 9.
- When you square an even number the result will be divisible by 4 (one example of ‘proof’ shown below)



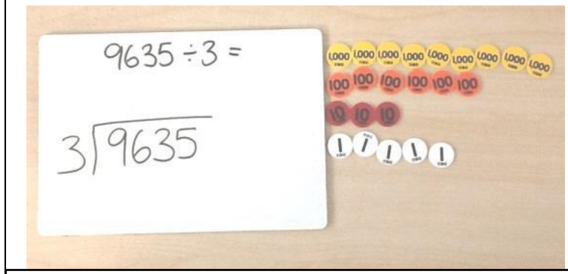
Key Questions
What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?
What is the inverse?
How can you use your multiplication and division facts to support you?

Year 5

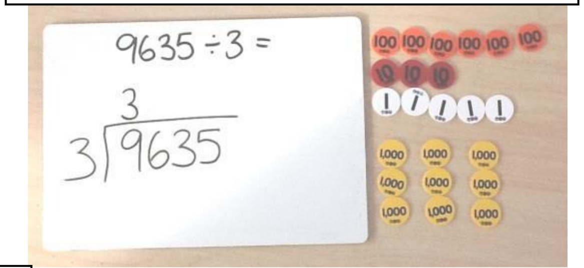
Methods - Concrete, Pictorial, Abstract (CPA)	**Concrete**	**Pictorial**	**Abstract**

Short division with remainders

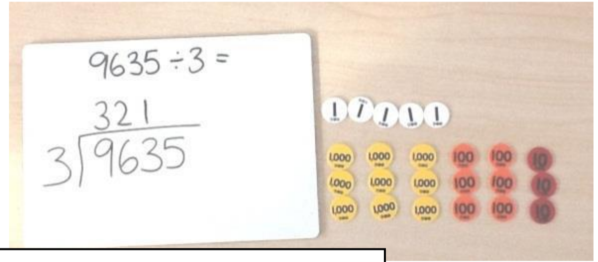
Create the dividend using Place Value counters.



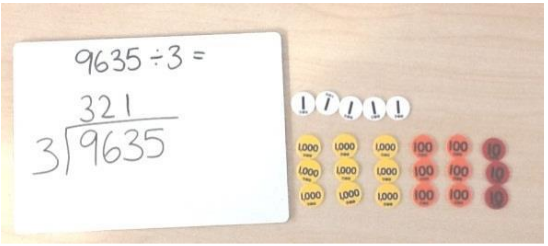
Group the 1000s counters according to the divisor and write the number of groups above the line in the thousands column.



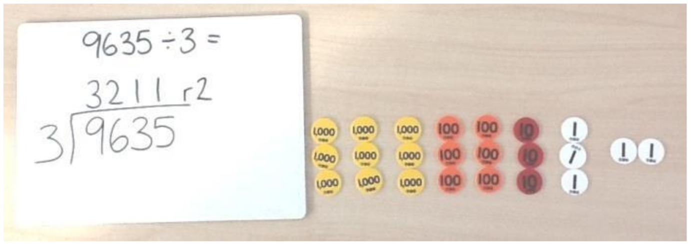
Group the 10s counters according to the divisor and write the number of groups above the line in the tens column.



Group the 10s counters according to the divisor and write the number of groups above the line in the tens column.



Group the ones counters according to the divisor and write the number of groups about the line in the ones column. Express remainders as 'r2' as part of the quotient.



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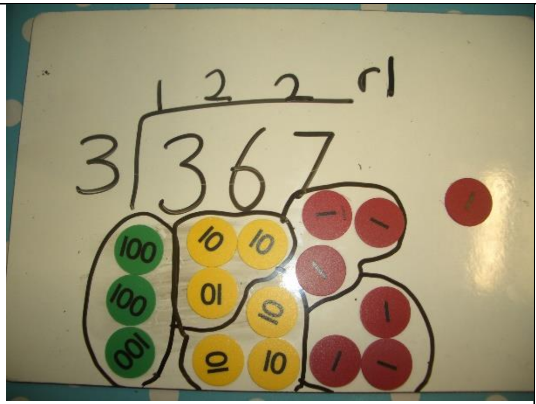
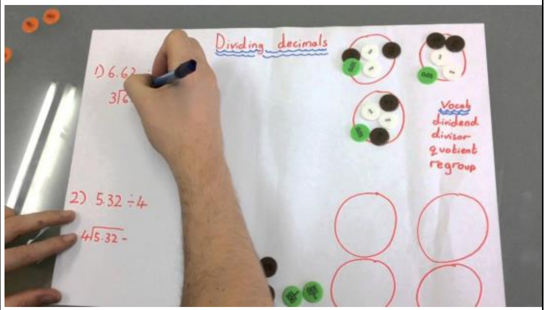
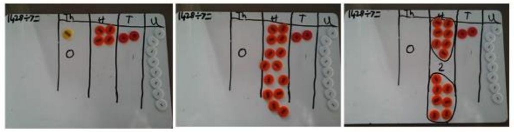
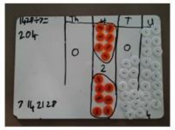


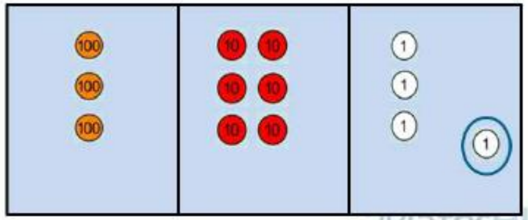


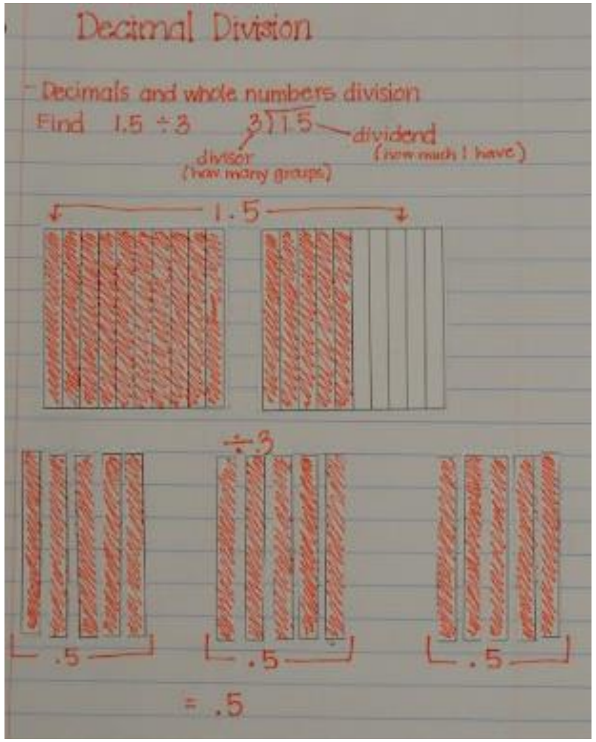
1428 ÷ 7 =

Children used the place value counters to illustrate their understanding of division with larger numbers
</td>
<td>
364 ÷ 3 =

$$3\overline{)\,3\,6\,4}\quad \text{121 rem 1}$$



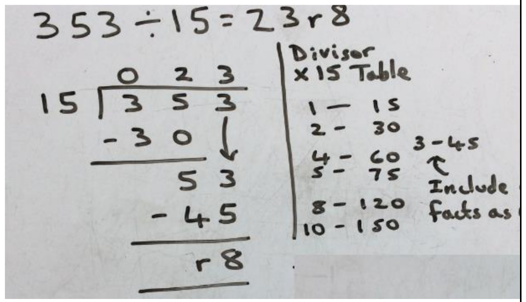


</td>
<td>
Move onto divisions with a remainder. Once children understand remainders, begin to express as a fraction or decimal according to the context.

$$\begin{array}{r} 8\ \ 6\ \ \text{r}\ 2 \\ 5\overline{)\,4\ \ {}^{3}\!3\ \ 2} \end{array}$$

$$\begin{array}{r} 1\ \ 8\ \ 6\ 1/5 \\ 5\overline{)\,9\ \ {}^{4}3\ \ {}^{3}1} \end{array}$$

$$\begin{array}{r} 1\ \ 4\ .\ 6 \\ 35\overline{)\,5\ \ {}^{16}1\ \ 1\ .\ {}^{21}0} \end{array}$$



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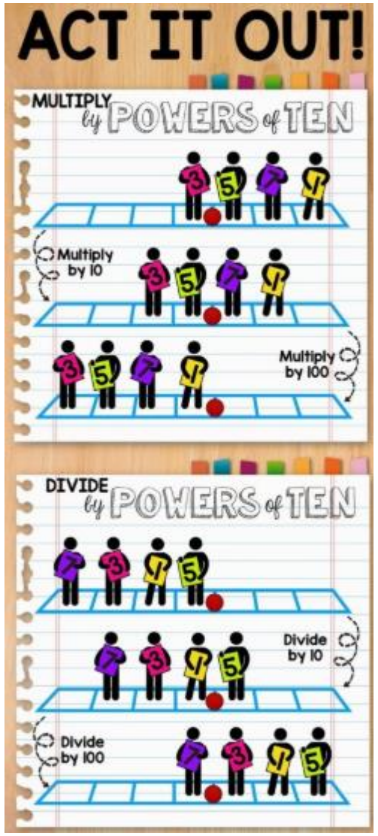
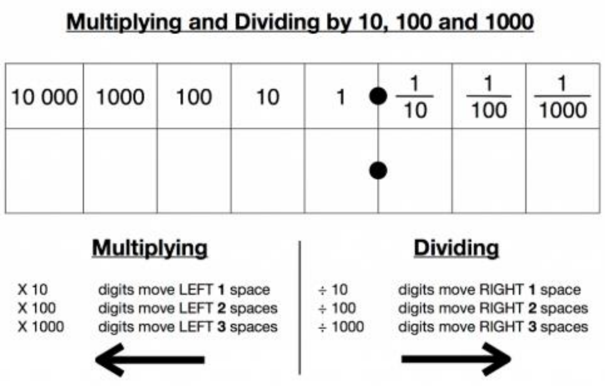
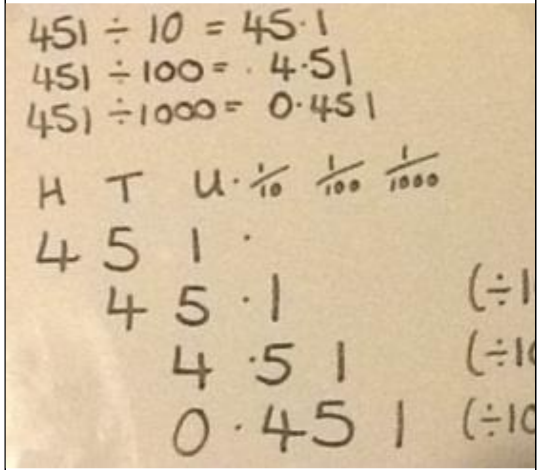
<tr>
<td>Divide whole numbers and those involving decimals by 10, 100 and 1000 by moving the digits around the fixed decimal.</td>
<td>Children may act it out so they gain a physical understanding of how the digits move and decimal stays in the same place.



Children may find using number and symbol cards and physically moving the digits to support their understanding.

</td>
<td>

Children can use number cards and the place value board above. Or they may write the numbers on the board to support their understanding.</td>
<td>

Children will move onto writing the calculations and solve the division statements without using a place value board or number cards. They still may use jottings to support them where appropriate.</td>
</tr>
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Year 6

Division Learning Objectives

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the 4 operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

Guidance (non-statutory)

- Pupils practise addition, subtraction, multiplication and division for larger numbers, using the formal written methods of columnar addition and subtraction, short and long multiplication, and short and long division.
- They undertake mental calculations with increasingly large numbers and more complex calculations.
- Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency.
- Pupils round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50, etc, but not to a specified number of significant figures.
- Pupils explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9.
- Common factors can be related to finding equivalent fractions.

Mental Strategies

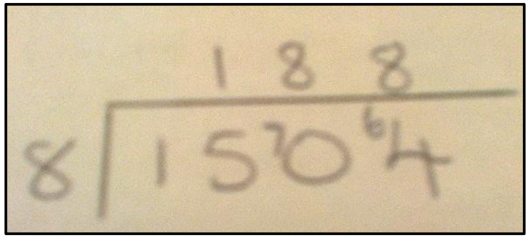
÷ = signs and missing numbers
Continue using a range of equations but with appropriate numbers

Sharing and Grouping and using a number line
Children will continue to explore division as sharing and grouping, and to represent calculations on a number line as appropriate.
Quotients should be expressed as decimals and fractions

Formal Written Calculations and Strategies

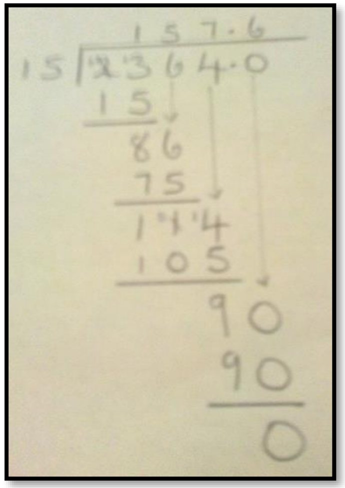
Short division.

1504 ÷ 8



Long Division (see below for more examples and clear outline of strategies used).

2364 ÷ 15



<u>**Generalisation – identifying patterns**</u>

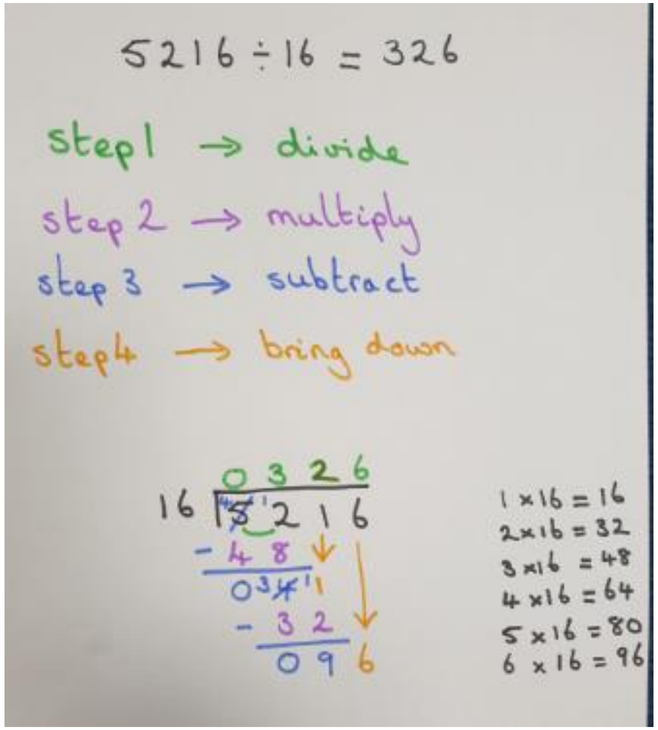
Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as BODMAS, or could be encouraged to design their own ways of remembering.

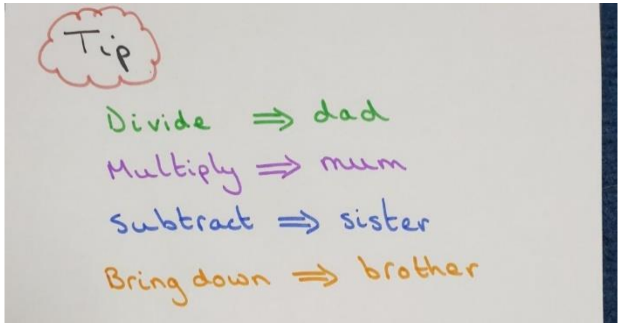
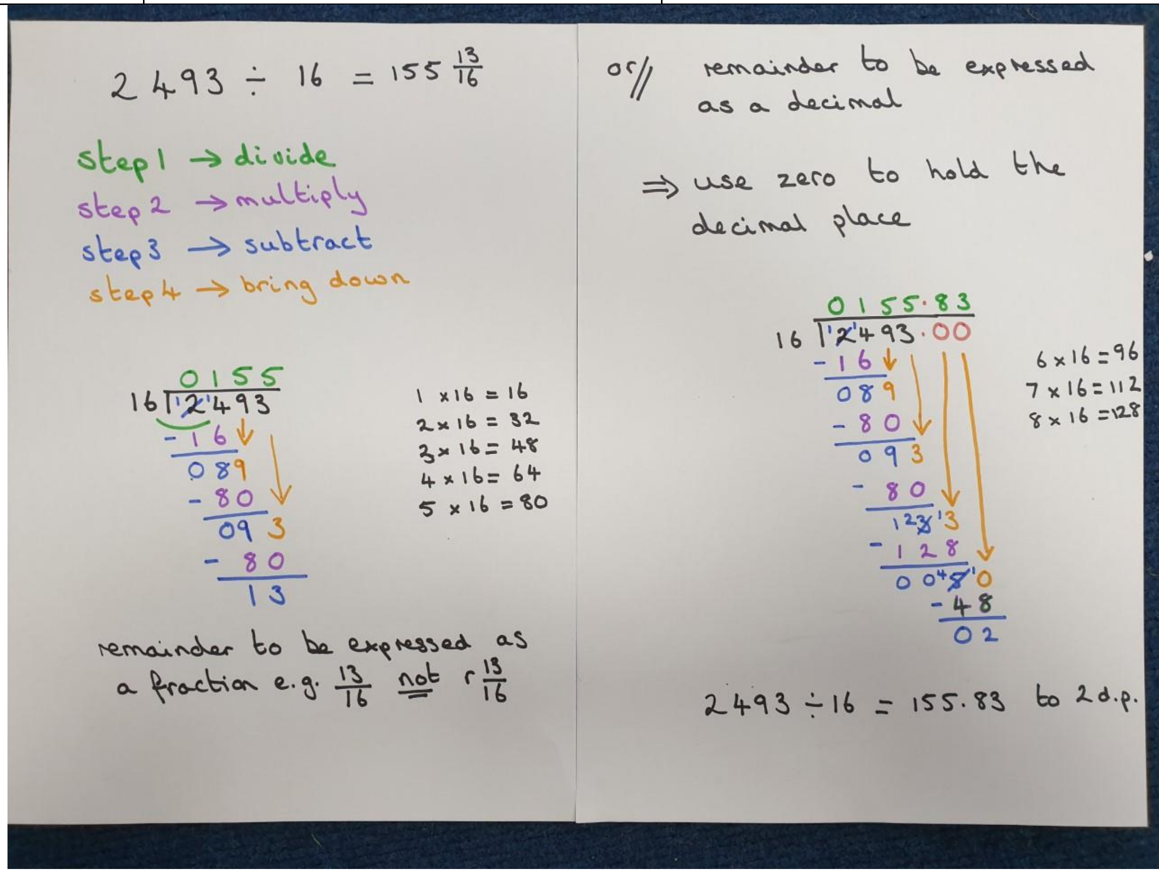
Sometimes, always, never true questions about multiples and divisibility. E.g.: If a number is divisible by 3 and 4, it will also be divisible by 12. (also see year 4 and 5, and the hyperlink from the Y5 column)

Using what you know about rules of divisibility, do you think 7919 is a prime number? Explain your answer.

<u>**Key Questions**</u>

What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?
What is the inverse?
How can you use your multiplication and division facts to support you?

<table>
<tr><th>Methods - Concrete, Pictorial, Abstract (CPA)</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr><td>Long Division</td><td>


</td><td>
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