

Katherine Semar Addition Calculation Policy

Progression through Calculations for Addition

Addition
EYFS
Composition of number is introduced through practical games, activities and oral stories. Children learn to combine two or more numbers using a range of concrete apparatus to understand how parts of a number become a whole. Using talk and practical apparatus, children will understand that addition can be carried out in any order to obtain the same total. Through observing and discussing a range of concrete and then pictorial patterns of numbers (e.g. tens frames), children will begin to instantly recognise the parts and mentally combine them to recognise the whole. They will be encouraged to use this conceptual subitising to recall key number facts.
Children are further introduced to addition as augmentation (increasing) through practical games, activities and oral stories. Children are encouraged to apply their number facts or count on (e.g. using a number track) in order to find the total.
Once children are secure with the concepts, teachers will introduce the signs + and = through arm gestures and modelling of the abstract calculation. Children are encouraged to represent their calculation using mathematical graphics and, if appropriate, the abstract form.
Links to ELG:
Have a deep understanding of numbers to ten, including the composition of each number. Automatically recall number bonds up to five and some bonds to ten including double facts.

Methods - Concrete, Pictorial, Abstract (CPA)	<u>Concrete</u>	<u>Pictorial</u>	<u>Abstract</u>
Addition by combining (aggregation)			
		Tell your partner about the birds on the trees. How many birds on the first tree? How many birds on the second tree? How many birds altogether? Children may draw their own pictorial representations of an addition problem.	
	There are three bananas in the green bowl and four bananas in the blue bowl. How many bananas all together? Progress from identical to non-identical objects.	State State	
		There are three frogs in the pond and two frogs on a log. How many frogs altogether?	

		in and 2" is and 2" is and 2" in house " in house " in feeling	
Conceptual Subitising	Counters used to create patterns which support conceptual subitising.	I know it is five because I see 3 and 1 and 1. I know it is five because I see 3 and 2.	

Addition by increasing (augmentation)

Oral addition stories with real life objects



First there were two frogs on a log



Then three more frogs came along



Now there are 5 frogs on the log Children can place real life objects along a number track as they tell the story.



First there are four frogs on a log. Then one more hops along to join them on the log. Now there are...?

Children may draw their own pictorial representations of the addition story.

Addition Learning Objectives

- read, write and interpret mathematical statements involving addition (+) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add one-digit and two-digit numbers to 20, including 0
- solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as 14 = ? + 9

Mental Strategies

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10. Children should memorise and reason with number bonds for numbers to 20, experiencing the = sign in different positions. They should see addition and subtraction as related operations. E.g. 7 + 3 = 10 is related to 10 - 3 = 7, understanding of which could be supported by an image like this.



Use bundles of straws and Dienes to model partitioning teen numbers into tens and ones and develop understanding of place value. Children have opportunities to explore partitioning numbers in different ways.

e.g. 7 = 6 + 1 7 = 5 + 2 7 = 4 + 3 = ?

Children should begin to understand addition as combining groups and counting on.



Formal Written Calculations and Strategies	
+ = signs and missing numbers Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'. 2 = 1 + 1 2 + 3 = 4 + 1	
Missing numbers need to be placed in all possible places. $3 + 4 = \Box$ $3 + \Box = 7$ $7 = \Box + 4$	
Counting and Combining sets of Objects Combining two sets of objects (aggregation) which will progress onto adding on to a set (augmentation)	
Understanding of counting on with a number track.	
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15	
Understanding of counting on with a number line (supported by models and images). 7+4 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 +	
 <u>Generalisation – identifying patterns</u> True or false? Addition makes numbers bigger. True or false? You can add numbers in any order and still get the same answer. 	
(Links between addition and subtraction) When introduced to the equals sign, children should see it as signifying equality. They should become used to seeing it in different positions:	

24 = 20 + 4 10 + 10 = 12 + 8 9 = 3 + 3 + 3 8 + 3 = 7 + ?

Key Questions

How many altogether? How many more to make...? I add ...more. What is the total? How many more is... than...? How much more is...? One more, two more, ten more... What can you see here?

Is this true or false?

What is the same? What is different?

<u>Methods - Concrete,</u> <u>Pictorial, Abstract (CPA)</u>	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model.	Use part part whole model. Use cubes to add two numbers together as a group or in a bar.	3 yerr yerr 2 yerr 2 yerr 2 yerr 2 yerr yer	4 + 3 = 7 5 3 $10 = 6 + 4$ Use the part-part whole diagram as shown above to move into the abstract.
Represent and use number bonds and related subtraction facts within 20.	Begin by using real-life objects in a context for children to understand. Then move onto representing with concrete objects such as a tens frame and counters, coloured cubes, Numicon, etc. $\boxed{6 + 4} = 10$	Put 5 cupcakes on two plates. part yhole 5 part 2 and 3 make 5. This is a number bond. 3 3 3 3 3 3 3 3	Fact Families 3 + 7 = 10 7 + 3 = 10 10 - 7 = 3 10 - 3 = 7 Missing numbers 9 + ? = 10 8 = 10 - ? Continue the pattern 10 + 8 = 18 11 + 7 = 18 Can you make up a similar pattern for the number 17?

		Children draw their own representations to support them e.g. drawing counters.	
Starting at the largest number and counting on.	Start with the larger number on the bead string and then count on to the smaller num- ber 1 by 1 to find the answer.	12 + 5 = 17 10 11 12 13 14 15 16 17 18 19 20 Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10. This is an essential skill for column addition later.	6 + 5 = 11 Start with the bigger number and use the smaller number to make 10. Use ten frames.	Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10. 9 + 5 = 14	7 + 4 = 11 If have seven pencils, how many more do I need to make 10. How many more do I add on now?
Add one-digit and two-digit numbers to 20, including zero.	2 more than 5.	Draw 2 more hats $5+2 =$	Emphasis should be on the language: '1 more than 5 is equal to 6.' '2 more than 5 is 7.'

Addition Learning Objectives

- using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods
- recall and use addition facts to 20 fluently, and derive and use related facts up to 100
- add numbers using concrete objects, pictorial representations, and mentally, including:
 - a two-digit number and 1s
 - a two-digit number and 10s
 - 2 two-digit numbers
 - adding 3 one-digit numbers
- show that addition of 2 numbers can be done in any order (commutative)
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

Mental Strategies

Children should count regularly, on and back, in steps of 2, 3, 5 and 10. Counting forwards in tens from any number should lead to adding multiples of 10. Number lines should continue to be an important image to support mathematical thinking, for example to model how to add 9 by adding 10 and adjusting. +10



Children should practise addition to 20 to become increasingly fluent. They should use the facts they know to derive others, e.g. using 7 + 3 = 10 to find:

17 + 3= 20

70 + 30 = 100

They should use concrete objects such as bead strings and number lines to explore missing numbers 45 + __ = 50.

As well as number lines, 100 squares could be used to explore patterns in calculations such as **74 +11**, **77 + 9** encouraging children to think about 'What do you notice?' where partitioning or adjusting is used.

Children should learn to check their calculations, by using the inverse.

They should continue to see addition as both combining groups and counting on.

They should use Dienes to model partitioning into tens and ones and learn to partition numbers in different ways e.g.

23 = 20 + 3 = 10 + 13.

Formal Written Calculations and Strategies



Leading to exchanging:

72

//////:

Expanded written method

40 + 7 + 20 + 5 = 40+20 + 7 + 5 = 60 + 12 = 72 40 + 7+ 20 + 560 + 12 = 72

Generalisation – identifying patterns

- Noticing what happens when you count in tens (the digits in the ones column stay the same)
- Odd + odd = even; odd + even = odd; etc.
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.



Key Questions

How many altogether? How many more to make...? How many more is... than...? How much more is...? Is this true or false? If I know that 17 + 2 = 19, what else do I know? (e.g. 2 + 17 = 19; 19 - 17 = 2; 19 - 2 = 17; 190 - 20 = 170 etc). What do you notice? What patterns can you see?

Methods - Concrete, Pictorial, Abstract (CPA)	<u>Concrete</u>	<u>Pictorial</u>	Abstract
Adding multiples of ten.	40 + 10 = 50 Model using dienes and bead strings Moving to the more abstract using place value counters and place value boards.	3 tens + 5 tens = tens 30 + 50 = Use representations for base ten.	20 + 30 = 50 70 = 50 + 20 $40 + \Box = 60$
Use known number facts Part-Part-Whole.	Children explore ways of making numbers within 20.		20
Using known facts.	+	Draw representations of calculations: $ \begin{array}{ccccccccccccccccccccccccccccccccccc$	Making connections $3 + 4 = 7$ between numberleads tosentences and numberleads tofacts. $30 + 40 = 70$ leads to $300 + 400 = 700$
Bar model and Part-Part- Whole model.	Real life objects 7 + 3 = 10	7 + 3 = 10	7 3 ? 7+3=10





Addition Learning Objectives

- add numbers mentally, including:
 - a three-digit number and 1s
 - a three-digit number and 10s
 - a three-digit number and 100s
- add numbers with up to 3 digits, using formal written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition number facts, place value, and more complex addition
- Add and subtract fractions with the same denominator within one whole.

Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged. This will help to develop children's understanding of working mentally.

Children should continue to partition numbers in different ways.

They should be encouraged to choose the mental strategies which are most efficient for the numbers involved, e.g.

Add the nearest multiple of 10, then adjust such as 63 + 29 is the same as 63 + 30 - 1;

counting on by partitioning the second number only such as

72 + 31 = 72 + 30 + 1 = 102 + 1 = 103

Manipulatives can be used to support mental imagery and conceptual understanding. Children need to be shown how these images are related e.g. What's the same? What's different?



Formal Written Calculations and Strategies

Missing number problems using a range of equations as in Year 1 and 2 but with appropriate, larger numbers.

Partition into tens and ones

Partition both numbers and recombine.

Count on by partitioning the second number only e.g.

247 + 125 = 247 + 100 + 20+ 5

= 347 + 20 + 5

= 367 + 5

= 372

Children need to be secure adding multiples of 100 and 10 to any three-digit number including those that are not multiples of 10.

Towards a Written Method

Introduce expanded column addition modelled with place value counters (Dienes or place value counters could be used for those who need a less abstract representation)



Leading to children understanding the exchange between tens and ones.



Children will use a formal columnar algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method.

247
+125
372
10

Generalisation – identifying patterns

Noticing what happens to the digits when you count in tens and hundreds. Odd + odd = even etc (see Year 2) Inverses and related facts – develop fluency in finding related addition and subtraction facts. Develop the knowledge that the inverse relationship can be used as a checking method.

Key Questions

What do you notice? What patterns can you see?

When comparing two methods alongside each other: What's the same? What's different? Look at this number in the formal method; can you see where it is in the expanded method / on the number line?

442 + 223 = $+ \frac{400 + 40 + 2}{200 + 20 + 3} = 665$ + 200 642 665 662 442







Addition Learning Objectives

- add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition two-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving increasingly harder fractions to calculate quantities
- add and subtract fractions with the same denominator
- compare numbers with the same number of decimal places up to two decimal places
- solve simple measure and money problems involving fractions and decimals to two decimal places
- solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number
- add and subtract fractions with the same denominator

Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100. The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate. Children should continue to partition numbers in different ways.

They should be encouraged to choose from a range of strategies:

- Counting forwards and backwards: 124 47, count back 40 from 124, then 4 to 80, then 3 to 77
- Reordering: 28 + 75, 75 + 28 (thinking of 28 as 25 + 3)
- Partitioning: counting on or back: 5.6 + 3.7, 5.6 + 3 + 0.7 = 8.6 + 0.7
- Partitioning: bridging through multiples of 10: 6070 4987, 4987 + 13 + 1000 + 70
- Partitioning: compensating 138 + 69, 138 + 70 1
- Partitioning: using 'near' doubles 160 + 170 is double 150, then add 10, then add 20, or double 160 and add 10, or double 170 and subtract 10
- Partitioning: bridging through 60 to calculate a time interval What was the time 33 minutes before 2.15pm?
- Using known facts and place value to find related facts.

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

Formal Written Calculations and Strategies

Place value learning from previous years to be revisited in order to consolidate understanding as a key to progressing learning.

Written methods (progressing to 4-digits)

Expanded column addition modelled with place value counters, progressing to calculations with 4-digit numbers.

<u>1000 + 200 + 50 + 3</u>	= 4000 + 200 + 110 + 6	1253	Thousands	Hundreds	Tens	Ones	
3000 + 0 + 60 + 3		<u>+ 3063</u>	1,000	100 100	10 10 10	000	1253
		6			10 10		1200
		110					+3063
		200			10 10 10	000	
		4000	1,000 1000 1000		10 10 10		
		4316					

Compact written method

Extend to numbers with at least four digits.



Children should be able to make the choice of reverting to expanded methods if experiencing any difficulty.

Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits).

72.8

<u>Generalisation – identifying patterns</u>

Investigate when re-ordering works as a strategy for addition.

Sam calculates that 263 + 491 = 754. Can you use subtraction to prove that he is correct?

Application of prior knowledge to solve other calculations e.g.

If I know that 70 + 30 = 100, I know that 700 + 300 = 1000, therefore 1700 + 300 = 2000, then I can apply this to solve 2700 + 3300 = 6000, etc.

Key Questions

What do you notice? What's the same? What's different? Can you convince me? How do you know? Is there another approach or method you could apply/use? Do you agree/disagree? Explain your reasoning.

<u> Methods - Concrete,</u>	<u>Concrete</u>	Pictorial	<u>Abstract</u>
Pictorial, Abstract (CPA)			
Year 4	Make both numbers on a place value grid. Add up	Children can draw a pictorial representations of the	Formal written methods are used to record and
	the units/ones, tens, hundreds, thousands and	columns and place value counters to further support	solve addition problems up to a 4 digit number
Add numbers with up to 4		their learning and understanding.	and a 4 digit number with exchanging (where
digits.	exchange where appropriate.	NB Addition of money needs to have £ and p added	appropriate).
	As children move on to decimals, real money and	separately.	
	decimal place value counters can be used to		
	support learning.		





Addition Learning Objectives

- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- add and subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
- add and subtract fractions with the same denominator and denominators that are multiples of the same number
- solve problems involving number up to three decimal places
- recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal
- solve problems which require knowing percentage and decimal equivalents of 21, 41, 51, 52, 54 and those fractions with a denominator of a multiple of 10 or 25
- recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, 5 2 + 5 4 = 5 6 = 1 5 1]
- add and subtract fractions with the same denominator and denominators that are multiples of the same number

Mental Strategies

Children should continue to count regularly, on and back, now including steps of powers of 10. The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate. Children should continue to partition numbers in different ways.

They should be encouraged to choose from a range of strategies:

- Counting forwards and backwards in tenths and hundredths: 1.7 + 0.55
- Reordering: 4.7 + 5.6 0.7, 4.7 0.7 + 5.6 = 4 + 5.6
- Partitioning: counting on or back 540 + 280, 540 + 200 + 80
- Partitioning: bridging through multiples of 10:
- Partitioning: compensating: 5.7 + 3.9, 5.7 + 4.0 0.1
- Partitioning: using 'near' double: 2.5 + 2.6 is double 2.5 and add 0.1 or double 2.6 and subtract 0.1
- Partitioning: bridging through 60 to calculate a time interval: It is 11.45. How many hours and minutes is it to 15.20?
- Using known facts and place value to find related facts.

<u>Mental methods</u> should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. Children should practise with increasingly large numbers to aid fluency e.g. 12462 + 2300 = 14762

Formal Written Calculations and Strategies

Pupils to identify the place value in large whole numbers. Place value from previous years must be revisited to consolidate understanding in order to progress learning.

Written methods (progressing to more than 4-digits)

As year 4, progressing when understanding of the expanded method is secure, children will move on to the formal columnar method for whole numbers and decimal numbers as an efficient written algorithm.

172.83

+ 54.68

227.51

1 1 1

Place value counters can be used alongside the columnar method to develop understanding of addition with decimal numbers.

<u>Generalisation – identifying patterns</u>

Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9. What do you notice about the differences between consecutive square numbers?

Key Questions

What do you notice? What's the same? What's different? Can you convince me? How do you know? Is there another approach or method you could apply/use? Do you agree/disagree? Explain your reasoning.

Methods - Concrete,	Concrete	<u>Pictorial</u>	Abstract
Pictorial, Abstract (CPA)			
Year 5 - Add numbers with more than 4 digits. - Add decimals with 2 decimal places, including money.	Represent		
Fractions - common denominators	Children to physically make the pictorial reprentation using fraction cards. For example give them the pictorial representation to the right and 16 equal fraction cards to place over the representation.		$\frac{1}{16} + \frac{1}{4}$ $\frac{1}{16} + \frac{4}{16} = \frac{5}{16}$ $\frac{1}{16} + \frac{1}{4} = \frac{5}{16}$

Addition Learning Objectives

- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- (consolidate previous years objectives)

Mental Strategies

Consolidate previous years.

Children should experiment with order of operations, investigating the effect of positioning the brackets in different places.

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

$\frac{1}{2} + \frac{2}{4} = 1$



Formal Written Calculations and Strategies

Written methods

As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured.

Continue calculating with decimals, including those with different numbers of decimal places

Problem Solving

Teachers should ensure that pupils have the opportunity to apply their knowledge in a variety of contexts and problems (exploring cross curricular links) to deepen their understanding.

Generalisation – identifying patterns

Order of operations: brackets first, then multiplication and division (of equal importance, left to right) before addition and subtraction (of equal importance, left to right). Children could learn an acrostic such as BODMAS, or could be encouraged to design their own ways of remembering. Sometimes, always or never true?

Key Questions

What do you notice? What's the same? What's different? Can you convince me? How do you know? Is there another approach or method you could apply/use? Do you agree/disagree? Explain your reasoning. Why? Prove it?

<u>Methods - Concrete,</u> Pictorial, Abstract (CPA)	<u>Concrete</u>	<u>Pictorial</u>	Abstract
Year 6 - add several numbers of increasing complexity. - Including adding money, measure and decimals with different numbers of decimal points and mixed numbers and how to express answers.	As year five	As Year five	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
Add fractions and mixed numbers with different denominators using the concept of equivalent fractions. To ensure parity with subtraction of mixed numbers, always convert to improper fractions. Encourage pupils to express answers as mixed numbers rather that improper fractions.	Children can use strips of paper to make bar models to help them find equivalent fractions and support their understanding e.g. $\frac{3}{4} = \frac{6}{8}$ They then can progress onto drawing bar models or pictorial representions (as to the right) to support their understanding.	(+) $(+)$ $(+$	$\frac{3}{4} + \frac{7}{8} = \frac{5}{8}$ $\frac{1}{3}\frac{1}{4} = \frac{5}{8}$ $\frac{5}{8} + \frac{7}{8} = \frac{13}{8} = \frac{5}{8}$ $\frac{15}{8} = \frac{5}{8}$